WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

STSADSE05T-STATISTICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer any *four* questions from Q. No. 1-6 and any *two* questions from Q. No. 7-9

1. Find a function whose first difference is $2x^2+3x+1$ taking the interval of differencing as 1. 5

2. Show that 5

$$xf(a)+x^2f(a+h)+x^3f(a+2h)+\cdots$$
$$=\left(\frac{x}{1-x}\right)f(a)+\left(\frac{x}{1-x}\right)^2\Delta f(a)+\left(\frac{x}{1-x}\right)^3\Delta^2 f(a)+\cdots$$

3. If $\int_0^m f(x)dx = a_1f(x_1)+a_2f(x_2)+\cdots+a_nf(x_n)$, where $f(x)$ is the polynomial in x of $(n-1)$ th degree $(n\geq 6)$, prove that $5\sum_{r=1}^{n}a_rx_r^4 = m^5$. 5

4. Write an algorithm to find a real root of the equation $f(x)=x^3-x-1=0$ using the method of bisection. 5

5. Write down the algorithm to approximate $\int_{-\infty}^{\infty}\int_{-4}^{5}(x+y)x^2e^{-y^2}dxdy$. 5

(You are only permitted to draw random samples from $U(0,1)$ only as many times as you want.)

6. Suppose that the equation $x^2+cx+d=0$ has 2 real roots α and β. Show that the iteration process $x_{n+1}=-\frac{cx_n+d}{x_n}$ always converges to that root which has higher absolute value. 5

7. (a) Give the expression of the error term in any interpolation formula. 5+5

   (b) Using part (a), give the expression of the error term in Trapizoidal rule in the context of numerical integration.

8. (a) Show that $n!$ can be approximated as $\sqrt{2\pi}e^{-n}n^{n+\frac{1}{2}}$ for large n. 5+5

   (b) Using part (a) or otherwise show that $P(X = \alpha n)$ is approximately $\frac{1}{\gamma^n\sqrt{2n\pi\alpha(1-\alpha)}}$ for large n, where $X \sim Bin(n, 0.5)$ and α is a rational number and $\gamma = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

9. Describe any two methods of Monte Carlo integration to approximate the value of $\frac{\pi}{4}$. Justify, which one of these methods is better than the other. (3+3)+4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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