WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

PHSADSE04T-PHYSICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No.1 and any *two* questions from the rest

1. Answer any ***fifteen*** questions from the following: $2\times15 = 30$

(a) $x + y = 10$ is defined in natural number N. Show that this relation is not *transitive*.

(b) Show that the centre of a group, $Z(G)$, forms a subgroup.

(c) Show that Inverse of an element in a subgroup is the inverse of the element of the group.

(d) Show that two left cosets of a subgroup H either coincide completely, or else have no elements in common at all.

(e) Show three cube roots of unity form an abelian group under multiplication.

(f) Give irreducible representation of SU(2) group.

(g) Prove that the group of order two is always cyclic.

(h) A random number $\tilde{x}$ is distributed uniformly between [0, 1]. Find its variance.

(i) A book of 1000 pages contains 20 typographical errors. What is the probability that there is at least one error in a single page?

(j) In a one dimensional random walk, the probability of moving one step in the right is p for each step. Find the probability of reaching the starting point by a random walker after taking $2n$ number of steps.

(k) Find the standard deviation of the uniform distribution $f(x) = \frac{1}{n}$; $(x = 0,\ 1,\ 2,\ \cdots,\ n)$.

(l) Let x be distributed in the Poisson form. If $P(x=1) = P(x=2)$; Find the expectation value.

(m) Four coins are tossed simultaneously. Find the probability of obtaining 2 heads and 2 tails.

(n) Show that total area under a normal curve is unity.

(o) State the condition when a binomial distribution can be approximated to normal distribution.

(p) Let a homomorphism $f:(G,\bullet)\to(H,\bullet)$. Show that $f(e_G)=e_H$, where e_I is the identity element of group *I*.

(q) State the nature of the equations:

(i) $4\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0$ (ii) $\frac{\partial^2 U}{\partial x^2}-2\frac{\partial^2 U}{\partial x\partial y}+\frac{\partial^2 U}{\partial y^2}=0$

(r) Find the solution of one dimensional Laplace's equation, where at the boundaries, the solution $\psi(x=0)=\psi(x=l)=0$.

(s) Determine the condition under which the following differential equation can be solved by the method of separation of variables:

$C_1\frac{\partial\phi(x,t)}{\partial t}+C_2\nabla^2\phi(x,t)+V(x,t)\phi(x,t)=0$, where C_1 and C_2 are constants.

(t) Show that the equation $\left[a^2\frac{\partial^2}{\partial x^2}-b^2\frac{\partial^2}{\partial y^2}\right]\phi(x,y)=0$ can be expressed as the product of two linear partial differential equations with real coefficients.

2. (a) A multiplication table of a Group consists 6 elements is given below 3+2+2

e	a_1	a_2	a_3	a_4	a_5
a_1	e	a_4	a_5	a_2	a_3
a_2	a_5	e	a_4	a_3	a_1
a_3	a_4	a_5	e	a_1	a_2
a_4	a_3	a_1	a_2	a_5	e
a_5	a_2	a_3	a_1	e	a_4

(i) Identify the two elements and 3 elements subgroups

(ii) Find the left cosets of any one 2 elements subgroup.

(iii) Show that three elements subgroup is the invariant subgroup.

(b) Show that the set $\{1,-1,i,-i\}$ forms a cyclic group for multiplication. Find its generator. 3

3. (a) Using the method of least squares, fit a straight line to the four points: 4

x	1	2	3	4
y	1.7	1.8	2.3	3.2

(b) Let f be a homomorphism from $G\to G'$. Denote by *K* the set of all elements of *G* which are mapped to the identity element of G', Then *K* forms an invariant subgroup of *G*. Prove it. 3

(c) Statistical average of some function $f(x)$ is defined as 3

$\langle f(x)\rangle=\sum_i f(x_i)P_i$. Show that $\left(\frac{d^k}{dt^k}\langle e^{tx}\rangle\right)_{t=0}=\langle x^k\rangle$

4. (a) Prove that Poisson distribution may be obtained as a limiting case of Binomial distribution. 3

(b) In a bolt factory, machines A_1, A_2 and A_3 manufacture respectively 25, 35 and 40% of the total. Of their output 5, 4 and 2% are defective bolts. A bolt is drawn at random and found it defective. What is the probabilities that it was manufactured by the machine A_1, A_2 or A_3? 4

(c) Deduce the expression for mean of a binomial distribution. 3

5. (a) Solve: $\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}=0$ given 4

$$T(0, y)=0\,;\; T(x, \infty)=0$$

$$T(a, y)=0\,;\; T(x, 0)=\sin\left(\frac{\pi x}{a}\right)$$

(b) Let U and V are two solutions of Laplace's equation. If both of them satisfy either Dirichlet or Neumann boundary condition, then show that they are at most differ by a constant otherwise identical. 3

(c) Show that following transformation forms a group under multiplication, 3

$$\begin{pmatrix} ct' \\ x' \end{pmatrix}=\begin{pmatrix} \cosh\Psi & \sinh\Psi \\ \sinh\Psi & \cosh\Psi \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix}$$

Ignore the fact that range of Ψ, $(-\infty, \infty)$ is not finite.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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