WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE04T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) If a and b are positive, prove that the equation $x^5 - 5ax + 4b = 0$ has three real roots or only one according as $a^5 >$ or $<$ b^4.

(b) Remove the second term of the equation $x^3 + 6x^2 + 12x - 19 = 0$ and solve it.

(c) Examine whether $x^4 - x^3 + x^2 + x - 1 = 0$ is a reciprocal equation.

(d) If α be a root of the equation $x^3 + 3x^2 - 6x + 1 = 0$, prove that the other roots are $\frac{1}{1-\alpha}$ and $\frac{\alpha - 1}{\alpha}$.

(e) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ be roots of the equation $x^n + nax + b = 0$, prove that
$$(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\cdots(\alpha_1 - \alpha_n) = n(\alpha_1^{n-1} + a).$$

(f) Find the remainder when the polynomial $f(x)$ is divided by $(x-\alpha)(x-\beta)$, $\alpha \neq \beta$.

(g) Form a biquadratic equation with real coefficients two of whose roots are $2i \pm 1$.

(h) If $\alpha(\neq 1)$ be any n^{th} root of unity, then prove that the sum $1 + 3\alpha + 5\alpha^2 + \cdots$ upto n^{th} term $= \frac{2n}{\alpha - 1}$.

2. (a) Show that if the roots of the equation $x^4 + x^3 - 4x^2 - 3x + 3 = 0$ are increased by 2, the transformed equation is a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 4

(b) Solve the equation $x^7 - 1 = 0$. Deduce that $2\cos\frac{2\pi}{7}$, $2\cos\frac{4\pi}{7}$, $2\cos\frac{8\pi}{7}$ are roots of the equation $t^3 + t^2 - 2t - 1 = 0$. 4

3. (a) If α is a special root of $x^{11} - 1 = 0$, prove that $(\alpha + 1)(\alpha^2 + 1)\cdots(\alpha^{10} + 1) = 1$. 4

(b) Applying Strum's theorem show that the equation $x^3 - 2x - 5 = 0$ has one positive real root and two imaginary roots. 4

4. (a) If the equation $x^4 - 4px^3 + 8x^2 + 1 = 0$ has a multiple root λ, prove that $3p = \dfrac{\lambda^2 + 3}{\lambda}$ and the only positive value of p is $\left(\dfrac{4}{3}\right)^{\frac{3}{4}}$. 4

(b) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has four real and unequal roots if $-11 < k < -8$. 4

5. (a) Find the condition that the roots of the equation $x^3 + 3Hx + G = 0$ may have three real and distinct roots. 4

(b) Find the upper limit of the real roots of the equation $x^4 - 5x^3 + 40x^2 - 8x + 24 = 0$. 4

6. (a) Applying Newton's theorem find the sum of 7th powers of the roots of the equation $x^3 + qx + r = 0$. 4

(b) Show that the cubes of the roots of the cubic $x^3 + ax^2 + bx + ab = 0$ are the roots of the cubic $x^3 + a^3x^2 + b^3x + a^3b^3 = 0$. 4

7. (a) Prove that the equation $(x+1)^4 = a(x^4 + 1)$ is a reciprocal equation if $a \neq 0$ and solve it when $a = -2$. 4

(b) Find the values of a for which the equation $ax^3 - 6x^2 + 9x - 4 = 0$ may have multiple roots and solve the equation in each case. 4

8. (a) If α be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0$, show that $\alpha = -\dfrac{8d}{3c}$. 4

(b) The equation $3x^4 + x^3 + 4x^2 + x + 3 = 0$ has four distinct roots of equal moduli. Solve it. 4

9. (a) If α, β, γ be the roots of the equation $x^3 - qx + r = 0$, find the equation whose roots are $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} - \dfrac{1}{\gamma^2}$, $\dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} - \dfrac{1}{\alpha^2}$, $\dfrac{1}{\gamma^2} + \dfrac{1}{\alpha^2} - \dfrac{1}{\beta^2}$. 4

(b) If $\alpha_1, \alpha_2, \cdots \alpha_n$ be the roots of the equation $x^n + \dfrac{x^{n-1}}{1!} + \dfrac{x^{n-2}}{2!} + \cdots + \dfrac{1}{n!} = 0$ and $S_r = \sum \alpha_1^r$, show that $S_r = 0$ for $r = 2, 3, \cdots n$ but $S_r \neq 0$ for $r = n+1, n+2, \cdots$ 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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