WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

STSACOR14T-STATISTICS (CC14)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* from the following questions $5\times 4 = 20$

1. If $\rho_{1j} = \rho_1$ for $j = 2, 3, \ldots, p$ and $\rho_{ij} = \rho_2$ for $i, j = 2, 3, \ldots, p$; $i \neq j$, then find the expression for $\rho_{1.23.\ldots p}$ in terms of ρ_1 and ρ_2. Find $\rho_{1.23.\ldots p}$ when $\rho_1 = 0$ and comment. 3+2

2. If $\boldsymbol{X} \sim N_p(\boldsymbol{0}, \Sigma)$, show that for $a_1, a_2, \ldots, a_p > 0$, 5

$$P(|X_1| \geq a_1, |X_2| \geq a_2, \ldots, |X_p| \geq a_p) \leq \frac{\sqrt{\frac{2}{\pi}}\left(\sum_{i=1}^{p}\sqrt{\sigma_{ii}}\right)}{\sum_{i=1}^{p} a_i},$$

where $\sigma_{ii} = \operatorname{var}(X_i)$.

3. Write down, in brief, the method of principal component analysis. Also, mention its use in real life. 5

4. If the dispersion matrix $\Sigma = ((\sigma_{ij})) = ((\sigma^2 \rho^{|i-j|}))_{i,\, j=1,2,3,4}$, then find the multiple correlation coefficient $\rho_{1.234}$ and the partial correlation coefficient $\rho_{12.34}$. 5

5. Define Wilcoxon signed-rank statistic. Show that it is distribution free under appropriate null hypothesis. Also, show that the null distribution of the statistic is symmetric. $1\frac{1}{2}+1\frac{1}{2}+2$

6. Describe, how you judge a given sample is random. 5

GROUP-B

Answer any *two* from the following questions $10\times 2 = 20$

7. (a) Suppose $\boldsymbol{X} = (X_1, X_2, \ldots\ldots, X_p)'$ has a multinomial distribution with parameters m and $\pi_1, \pi_2, \ldots\ldots, \pi_p$, where $\sum_{i=1}^{p} \pi_i = 1$. Find the mgf of $\boldsymbol{X}$. Hence show that $\boldsymbol{X}^{(1)} = (X_1, X_2, \ldots\ldots, X_r)'$ with $r < p-1$, is also multinomial. 3+2

(b) For the random vector $\boldsymbol{X}$ defined in Question 7(a), show that the probability $P(X_1 = x_1, \ldots\ldots, X_p = x_p)$ reaches its maximum if and only if $m\pi_i - 1 < x_i \le (m+p-1)\pi_i$, $i = 1, 2, \ldots\ldots, p$. 5

8. (a) Suppose $\boldsymbol{X} = (X_1, X_2, X_3)' \sim N_3(\mathbf{0}, \Sigma)$, where $\Sigma = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix}$. Show that: 7

(i) $P(X_1 > 0,\ X_2 > 0,\ X_3 > 0) = \dfrac{1}{8} + \dfrac{\sin^{-1}\rho_{12} + \sin^{-1}\rho_{13} + \sin^{-1}\rho_{23}}{4\pi}$

(ii) $1 + 2\rho_{12}\,\rho_{13}\,\rho_{23} \ge \rho_{12}^2 + \rho_{13}^2 + \rho_{23}^2$.

(b) Let $\boldsymbol{X} = (X_1, X_2, X_3)'$ be a 3-dimensional random vector with probability mass function 3

$$p(x) = \begin{cases} \dfrac{x_1\, x_2\, x_3}{72} & \text{if } \ x_1 \in \{1, 2\}\ ;\ x_2 \in \{1, 2, 3\}\ ;\ x_3 \in \{1, 3\} \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the marginal probability mass function (pmf) of X_2.

(ii) Find the conditional pmf of X_1 given $X_2 = 2$, $X_3 = 1$.

9. Suppose we have two independent continuous populations with distribution functions $F_1(x)$ and $F_2(x)$. Consider the testing problem

$H_0 : F_1(x) = F_2(x)\ \forall\, x \in \mathbb{R}$ vs. $H_1 : F_1(x) \ge F_2(x)$ with strict inequality for at least one x.

(a) Under the above setup, does Wilcoxon rank sum test applicable? Justify your answer. 2

(b) If your answer in (a) is negative, suggest appropriate model restriction under which Wilcoxon rank sum test is applicable. 2

(c) If the suggested model restriction in (b) is not practically valid then describe, in details, how will you carry out the test for testing H_0 vs. H_1 ? 6

Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

——×——