WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 4th Semester Examination, 2022

MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) In $\mathbb{Z}_{14}$, find the smallest positive integer n such that $n[6]=[0]$. 2

(b) Let $(G,*)$ be a group. If every element of G has its own inverse then prove that G is commutative. 2

(c) Let H be a subgroup of a group G. Show that for all $a\in G$, $aH=H$ if and only if $a\in H$. 2

(d) Check whether the relation ρ defined by $x\rho y$ if and only if $|x|=|y|$, is an equivalence relation or not on the set of integers $\mathbb{Z}$. Justify your answer. 2

(e) Show that the alternative group A_3 is a normal subgroup of S_3. 2

(f) Show that every cyclic group is abelien. 2

(g) Show that the ring of matrices $\left\{\begin{pmatrix}2a & 0\\ 0 & 2b\end{pmatrix} : a, b\in \mathbb{Z}\right\}$ contains divisors of zero and does not contain the unity. 2

(h) Let A and B be two ideals of a ring R. Is $A\cup B$ an ideal of R? Justify. 2

2. (a) A relation ρ on the set $\mathbb{N}$ is given by $\rho=\{(a,b)\in \mathbb{N}\times\mathbb{N} : a \text{ is a divisor of } b\}$. Examine if ρ is (i) reflexive, (ii) symmetric, (iii) transitive. 4

(b) If G is a group such that $(ab)^2=a^2b^2$ for all $a,b\in G$; then show that G is commutative. 4

3. (a) Let $A=\{1,2,3\}$. List all one-one functions from A onto A. 4

(b) Let G be a commutative group. Show that the set H of all elements of finite order is a subgroup of G. 4

4. (a) Let H be a subgroup of a group G. Show that the relation ρ defined on G by "$a\rho b$ if and only if $a^{-1}b\in H$" for $a,b\in G$ is an equivalence relation. 4

(b) Prove that the order of every subgroup of a finite group G is a divisor of the order of G. 4

5. (a) Prove that every group of order less than 6 is commutative. 4

(b) Let $(G, \circ)$ be a cyclic group generated by a. Then prove that a^{-1} is also a generator. 4

6. (a) Show that the intersection of two normal subgroups of a group G is normal in G. 4

(b) Show that if H be a subgroup of a commutative group G then the quotient group G/H is commutative. Is the converse true? Justify. 4

7. (a) Prove that an infinite cyclic group has only two generators. 4

(b) In the rings $\mathbb{Z}_8$ and $\mathbb{Z}_6$, find the following elements: 2+2

(i) the units and (ii) the zero divisors.

8. (a) Find all ideals of $\mathbb{Z}$. 4

(b) Let R be a commutative ring with 1. Then prove that R is a field if and only if R has no non-zero proper ideals. 4

9. (a) (i) Let S be a set with n elements. How many binary operations can be defined on S? Justify. 2+2

(ii) Let A and B be two sets with $|A| = 5$ and $|B| = 2$. How many surjective functions defined from A onto B? Justify.

(b) Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a (\neq 0) \in \mathbb{R} \right\}$. Show that G forms a group w.r.t. matrix multiplication. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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