WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

STSACOR08T-STATISTICS (CC8)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer any *four* questions from the Question No. 1-6 $5\times4=20$

1. Show that $\mathcal{N}(\theta,\ \theta)$ distribution with $\theta>0$, belongs to one-parameter exponential family (OPEF) of distributions. Hence suggest a complete-sufficient statistic based on a random sample of size n from the same distribution. 2+3

2. (a) Define Fisher's information $I_T(\theta)$ contained in a statistic T. What is its value if T is ancillary for the concerned family of distributions? 2+3

   (b) For two equivalent statistics T_1 and T_2, show that $I_{T_1}(\theta)=I_{T_2}(\theta)$ [you may assume continuous probability distributions for the statistics].

3. Provide an example (with justification) where the variance of the UMVUE of a parametric function is greater than the corresponding Rao-Cramer Lower Bound. 5

4. (a) Let $\phi(x)$ be a test function defined for testing $H_0:\theta\in\Theta_0$ *vs.* $H_1:\theta\in\Theta_1$ $(\subseteq\Theta\cap\Theta_0^c)$, where Θ is the associated parameter space. Distinguish between size and level of significance of above test. Also define the power function of the test. 3+2

   (b) Give an example of a test having $size=\frac{17}{40}$ and $power=\frac{23}{40}$.

5. Construct uniformly most powerful (UMP) test of size α for testing $H_0:\theta=\theta_0$ (known) *vs.* $H_1:\theta>\theta_0$ on the basis of a random sample of size n from exponential distribution with mean $\theta, \theta>0$. Is there any change in the optimality criteria of your test if we consider the null hypothesis $H_0':\theta\le\theta_0$ against the same alternative H_1? Justify your answer. 3+2

6. Show that most powerful test is necessarily unbiased. Provide an example of a biased test. 3+2

Answer any *two* questions from the Question No. 7-9 $10\times2 = 20$

7. (a) Let $x_1, x_2, ..., x_n$ be a random sample from $\mathcal{N}(\theta, 1)$ distribution with $\theta \in \mathbb{R}$. Based on the observations find a complete-sufficient statistic and an ancillary statistic. Make comment on the joint distribution of these two statistics. 6+4

   (b) Construct an example of two statistics T_1 and T_2 such that T_1 is not sufficient and T_2 is ancillary, but $(T_1, T_2)'$ is jointly sufficient for the associated family of distributions [state necessary result(s)].

8. (a) Define maximum likelihood $\theta \in \mathbb{R}$, estimator (mle). On the basis of a random sample of size $n = 2k+1$ from *Laplace* $(\theta, 1)$ distribution, find the mle of θ. 4+6

   (b) For *Laplace* $(\theta, 1)$ find Fisher's information contained in (the data) $\boldsymbol{X}$ and that contained in the mle of θ (obtained in (a)). Compare your findings.

9. (a) Describe likelihood ratio test (LRT). State its properties. 5+5

   (b) On the basis of two independent samples of sizes n_1 and n_2 from $\mathcal{N}(\mu_1, \sigma^2)$ and $\mathcal{N}(\mu_2, \theta^2\sigma^2)$ distributions respectively, obtain LRT of size α for testing $H_0: \theta = 1$ *vs.* $H_1: \theta > 1$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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