WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

PHSACOR08T-PHYSICS (CC8)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Prove the equivalence of the operators, $\dfrac{\partial}{\partial x}=\dfrac{\partial}{\partial z}+\dfrac{\partial}{\partial \bar{z}}$ and $\dfrac{\partial}{\partial y}=i\left(\dfrac{\partial}{\partial z}-\dfrac{\partial}{\partial \bar{z}}\right)$, where $z=x+iy$ and $\bar{z}=x-iy$.

(b) If $u(x,\, y)=2x(1-y)$, find a function $v(x,\, y)$ such that $f(z)=u+iv$ is analytic.

(c) Evaluate $\oint\limits_C \dfrac{e^z}{(z+4)^4}\,dz$ where C is the circle $|z|=3$ using Cauchy integral formula.

(d) Determine the nature of the singularities of $f(z)=\dfrac{z\,e^{iz}}{z^2+1}$ and evaluate the residues.

(e) Compute the integration $\int\limits_0^{1+i}(z^2-z)\,dz$ along the line $y=x$.

(f) Show that if $f(x)$ is an odd function then its Fourier transform is always an imaginary function.

(g) Find Fourier transform of $f'(t)=\dfrac{df}{dt}$ in terms of $\tilde{f}(\omega)$, where $\tilde{f}(\omega)$ is the Fourier transform of $f(t)$.

(h) Find the Fourier transform of the function $f(x)=\delta(x-a)+\delta(x+a)$ where $\delta(x)$ is the Dirac-delta function.

(i) Find the form of Laplace's equation in cylindrical co-ordinate starting from

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

(j) For $\mathbf{A}$, a $n\times n$ diagonal matrix show that $\det(e^{\mathbf{A}}) = e^{\mathrm{Tr}\mathbf{A}}$.

(k) If two matrices A and B are such that $AB = BA$, show that $AB^{-1} = B^{-1}A$.

(l) If $\mathbf{A}$ is an antisymmetric matrix and $\mathbf{A}^2 + \mathbf{I} = \mathbf{0}$, then show that $\mathbf{A}$ is orthogonal.

(m) Show that Hermitian matrix remains Hermitian under similarity transformation.

(n) Show that all the eigenvalues of a Hermitian matrix are real.

2\. (a) Show that $\oint_C \dfrac{dz}{z(z+1)} = \begin{cases} 0 & , \text{ for } R>1 \\ 2\pi i & , \text{ for } R<1 \end{cases}$ in which the contour C is the circle defined by $|z| = R$. — 3

(b) Show that the Fourier transform $\tilde{f}(k)$ of the function $f(x)$ given by — 3

$$f(x) = \begin{cases} 0 & , \quad -\infty < x < -a \\ 1 & , \quad -a < x < a \\ 0 & , \quad a < x < \infty \end{cases}$$

is $\tilde{f}(k) = \sqrt{\dfrac{2}{\pi}}\dfrac{\sin ka}{k}$.

(c) Find the eigenvalues and eigenvectors of the Hermitian matrix $\mathbf{H} = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. — 2+2

Construct a unitary matrix $\mathbf{U}$ such that $\mathbf{U}^\dagger \mathbf{H} \mathbf{U} = \mathbf{D}$, where $\mathbf{D}$ is a real diagonal matrix.

3\. (a) Solve the one-dimensional heat equation $\dfrac{\partial u(x,t)}{\partial t} = \dfrac{\partial^2 u(x,t)}{\partial t^2}$ subject to boundary conditions $u(0,t) = u(1,t) = 0$ and initial condition $u(x,0) = \sin(\pi x) + \sin(2\pi x)$ for $t > 0$. — 3

(b) If inner product between two matrices is defined by $(\mathbf{A}, \mathbf{B}) = \mathbf{Tr}(\mathbf{A}^\dagger \mathbf{B})$ then show that the matrices $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & 1 \\ 4 & 2 \end{pmatrix}$ are orthogonal. — 2

(c) Show that eigenvalues of an anti-Hermitian matrix is either zero or purely imaginary. — 2

(d) Show that $\displaystyle\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\, dx = \sqrt{\pi}$. — 3

4\. (a) Consider a hollow sphere of internal and external radius r_1 and r_2 maintained at temperatures T_1 and T_2 respectively. Find the temperature distribution inside the sphere. At what distance from the center the temperature will be the arithmetic mean of surface temperatures. — 4+2

(b) If A and B are two matrices and both commute with their commutator, then show that $\exp(A)\exp(B) = \exp(A + B + \frac{1}{2}[A, B])$. — 4

5. (a) The displacement of a damped harmonic oscillator as a function of time is given by 2+2

$$f(t) = \begin{cases} 0 & , \text{ for } t < 0 \\ e^{-t/s}\sin(\omega_0 t) & , \text{ for } t \geq 0 \end{cases}$$

Find the Fourier transform of this function and so give a physical interpretation of Parseval's theorem.

(b) Using Fourier transformation solve the following one dimensional wave equation 4

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \ , \quad u(x,\, 0) = f(x) \quad \text{and} \quad \frac{\partial u(x,\, 0)}{\partial t} = 0$$

(c) If $g(k)$ is the Fourier transform of $f(x)$, then show that $g(-k) = g^{*}(k)$ is the sufficient and necessary condition for $f(x)$ to be real. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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