



WEST BENGAL STATE UNIVERSITY
B.Sc. Honours/Programme 2nd Semester Examination, 2022

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any **five** questions from the following: 2×5 = 10
- (a) Test whether the equation $(\sin 2x - \tan y) dx = x \sec^2 y dy$ is exact or not?
- (b) Find an integrating factor of the differential equation $(2x^2 + y^2 + x) dx + xy dy = 0$.
- (c) Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.
- (d) Verify if the following pair of functions are independent
 e^x , $5e^x$
- (e) Given that $y_1(x)$, $y_2(x)$ and $y_3(x)$ are solutions of $\{D^2 + p(x)D + q(x)\}y = 0$, where $D \equiv \frac{d}{dx}$. Show that these solutions are linearly independent.
- (f) Verify the integrability of the following differential equation:
 $yz dx = zx dy + y^2 dz$
- (g) Determine the order, degree and linearity of the following P.D.E:
$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$$
- (h) Eliminate the arbitrary functions ϕ and ψ from $z = \phi(x + iy) + \psi(x - iy)$, where $i^2 = -1$.
2. (a) Determine the constant A of the following differential equation such that the equation is exact and solve the resulting exact equation: 4
$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right) dx + \left(\frac{1}{x^2} - \frac{1}{x}\right) dy = 0$$
- (b) Reduce the equation $\sin y \frac{dy}{dx} = \cos x (2 \cos y - \sin^2 x)$ to a linear equation and hence solve it. 4
3. (a) Using the transformation $u = x^2$ and $v = y^2$ to solve the equation 4
$$xyp^2 - (x^2 + y^2 - 1)p + xy = 0$$
, where $p = \frac{dy}{dx}$

(b) Solve: $\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$ 4

4. (a) Solve by the method of variation of parameters: 4

$$\frac{d^2y}{dx^2} + a^2y = \cos ax$$

(b) Show that e^x and xe^x are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0) = 1$, $y'(0) = 4$. Is it the unique solution? 1+1+1+1

5. (a) Solve: $\{(5+2x)^2 D^2 - 6(5+2x)D + 8\}y = 8(5+2x)^2$, where $D \equiv \frac{d}{dx}$. 4

(b) Solve the following equations: 4

$$\frac{dx}{dt} + 4x + 3y = t \quad ; \quad \frac{dy}{dt} + 2x + 5y = e^t$$

6. (a) Verify that the following equation is integrable, find its primitive: 5

$$zy dx + (x^2y - zx) dy + (x^2z - xy) dz = 0$$

(b) Solve: $(4x^2y - 6) dx + x^3 dy = 0$ 3

7. (a) Eliminate the arbitrary function ϕ from the relation $z = e^{my} \phi(x - y)$. 3

(b) Solve the PDE by Lagrange's method: 5

$$px(x + y) - qy(x + y) + (x - y)(2x + 2y + z) = 0$$

8. (a) Find the particular solution of the differential equation 4

$$(y - z) \frac{\partial z}{\partial x} + (z - x) \frac{\partial z}{\partial y} = x - y$$

which passes through the curve $xy = 4$, $z = 0$.

(b) Determine the points (x, y) at which the partial differential equation 4

$$(x^2 - 1) \frac{\partial^2 z}{\partial x^2} + 2y \frac{\partial^2 z}{\partial y \partial x} - \frac{\partial^2 z}{\partial y^2} = 0$$

is hyperbolic or parabolic or elliptic.

9. (a) Solve: $(x^2 + y^2 + z^2) dx - 2xy dy - 2xz dz = 0$ 4

(b) Solve in particular cases: 4

$$\frac{d^2y}{dx^2} + y = \sin 2x \quad ; \quad \text{when } x = 0, \quad y = 0 \quad \text{and} \quad \frac{dy}{dx} = 0$$

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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