



WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-II Examination, 2022

MATHEMATICS

PAPER: MTMA-IV

Time Allotted: 4 Hours

Full Marks: 100

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

GROUP-A

Answer any two questions from the following

10×2 = 20

1. (a) Find the image of the point $P(2, 3, 5)$ on the plane $5x + 8y - z + 16 = 0$. 5
- (b) Prove that the locus of polar of tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with respect to its auxiliary circle is $a^2x^2 + b^2y^2 = a^4$. 5
2. (a) Show that the feet of the normals from the point (α, β, γ) to the paraboloid $x^2 + y^2 = 2az$ lie on the sphere $x^2 + y^2 + z^2 - z(\alpha + \gamma) - \frac{y}{2\beta}(\alpha^2 + \beta^2) = 0$, $\beta \neq 0$. 5
- (b) Find the equation of the sphere touching the three coordinate planes. 5
3. (a) Find the equations of the tangent planes to the conicoid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the straight line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$. 5
- (b) Show that the perpendicular from the origin on the generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ lie on the surface $\frac{a^2(b^2 + c^2)^2}{x^2} + \frac{b^2(c^2 + a^2)^2}{y^2} = \frac{c^2(a^2 - b^2)^2}{z^2}$. 5

GROUP-B

Answer any one question from the following

10×1 = 10

4. (a) Find the eigenvalues and the corresponding eigenfunctions of the Boundary Value Problem $y'' + 2y' + (1 + \lambda)y = 0$, $y(0) = 0$, $y'(a) = 0$ 5
- (b) Solve: $\frac{dx}{dt} + 4x + 3y = t$; $\frac{dy}{dt} + 2x + 5y = e^t$ 5

5. (a) Find the equation of the integral surface given by the differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$, which passes through the circle $z=0$, $x^2 + y^2 = 2x$. 5
- (b) Using Charpit's method, find the complete integral of the equation $2(z + xp + yq) = yp^2$. 5

GROUP-C

Answer either Question No. 6 or Question No. 7 and either Question No. 8 or Question No. 9 13+12

6. (a) State and prove the fundamental theorem of Linear Programming. 6
- (b) Solve the L.P.P. : 7
- Maximize $z = 2x_1 + 3x_2 + x_3$
- Subject to $-3x_1 + 2x_2 + 3x_3 = 8$
- $-3x_1 + 4x_2 + 2x_3 = 7$
- $x_1, x_2, x_3 \geq 0$

OR

7. (a) By solving the dual of the following problem, show that the given problem has no feasible solution 7
- Minimize $z = x_1 - x_2$
- Subject to $2x_1 + x_2 \geq 2$
- $-x_1 - x_2 \geq 1$
- $x_1, x_2 \geq 0$

- (b) Solve graphically the following pay-off matrix: 6

		<i>B</i>	
		<i>B</i> ₁	<i>B</i> ₂
<i>A</i>	<i>A</i> ₁	2	-1
	<i>A</i> ₂	3	2
	<i>A</i> ₃	-1	5
	<i>A</i> ₄	-2	1

8. (a) Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix. 6

		MEN				
		I	II	III	IV	V
A	1	3	2	3	6	
B	2	4	3	1	5	
C	5	6	3	4	6	
D	3	1	4	2	2	
E	1	5	6	5	4	

(b) Solve the following transportation problem:

6

		TO			
		1	2	3	
FROM	I	5	1	7	10
	II	6	4	6	80
	III	3	2	5	15
		75	20	50	

OR

9. (a) Solve the travelling salesman problem given by the following data:

6

$$C_{12} = 20, \quad C_{13} = 4, \quad C_{14} = 10, \quad C_{23} = 5, \quad C_{34} = 6,$$

$$C_{25} = 10, \quad C_{35} = 6 \quad \text{and} \quad C_{45} = 20, \quad \text{where } c_{ij} = c_{ji}$$

and there is no route between cities i and j , if a value of c_{ij} be not shown above.

(b) Using dominance solve the game problem:

6

		B			
		-5	3	1	20
A	5	5	4	6	
	-4	-2	0	-5	

GROUP-D

Answer any *three* questions from the following

15×3 = 45

10.(a) A point is moving in a straight line with an acceleration μx towards a fixed centre in the straight line and with an additional acceleration $L \cos pt$ where μ , L and p are constants and x is the distance from the fixed point. If $p = \sqrt{\mu}$ and $x = a$, $\dot{x} = 0$ at $t = 0$ find x and \dot{x} at any time t .

7

(b) Find the motion of a projectile under constant gravity (resistance of air being neglected). Find the equation of its path, its time of flight and maximum horizontal range.

8

11.(a) A particle of unit mass is projected with velocity V at an angle α above the horizon in a medium whose resistance is k times the velocity of the particle. Show that the direction of its velocity will make an angle $\frac{\alpha}{2}$ above the horizon after time

8

$$\frac{1}{k} \log\left(1 + \frac{kV}{g} \tan \frac{\alpha}{2}\right)$$

(b) If t_1 and t_2 be the periods of the vertical oscillation of two different weights suspended by an elastic string and c_1 and c_2 be the statical extensions due to these weights, then prove that

7

$$g(t_1^2 - t_2^2) = 4\pi^2(c_1 - c_2)$$

12.(a) Find the radial and cross-radial components of velocity and acceleration of a particle moving in a plane curve. 8

(b) A particle moves under a force $m\mu\{3au^4 - 2a\alpha(a^2 - b^2)u^5\}$, $a > b$ and projected from an apse at a distance $a + b$ with a velocity $\frac{\sqrt{\mu}}{a + b}$, show that its orbit is $r = a + b \cos \theta$. 7

13.(a) A planet is describing an ellipse about the sun as focus; show that its velocity away from the sun is greatest when the radius vector to the planet is at right angle to the major axis of the path and it is $\frac{2\pi ae}{T\sqrt{1-e^2}}$ where $2a$ is the major axis, e the eccentricity and T the periodic time. 8

(b) If a body moves under central force F per unit mass moves in a medium whose resistance is $k(\text{velocity})^2$ per unit mass. Show that the Differential Equation of the orbit is 7

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h_0^2 u^2} e^{2ks},$$

h_0 is the initial angular momentum per unit mass about the centre of force.

14.(a) A particle starts from rest and slides from the highest point of a rough circle. Prove that it will leave the curve at a point θ given by 7

$$(1 + 2\mu^2)e^{-2\mu\theta} - \frac{1}{2}\cos\theta + \mu\cos\theta = 0$$

(b) An engine is pulling a train and works at a constant power, doing H units of work per second. If M is the mass of the whole train and F , the resistance (supposed constant) then prove that the time of generating velocity V from rest is 8

$$\left(\frac{MH}{F^2} \log \frac{H}{H - FV} - \frac{MV}{F} \right) \text{ seconds}$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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