WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2022

PHYSICS

PAPER: PHSA-V

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

UNIT-VA

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Explain giving examples the meaning of holonomic and non-holonomic constraints.

(b) What are cyclic co-ordinates? Give examples.

(c) Define Poisson Bracket and write down the Hamilton's equation of motion using Poisson Bracket.

(d) Which statistics will be applicable to deuterons and phonon?

(e) Simultaneous event as seen by one Lorentz frame observer will not look simultaneous when seen by a different Lorentz frame observer. Justify this statement on the basis of Lorentz transformation.

(f) Define microstates and macrostates corresponding to a microcanonical ensemble.

(g) What is Fermi momentum? Why is it non-zero even at temperature $T = 0$?

(h) "Condensation from vapour to liquid occurs in the co-ordinate space. In contrast, Bose-Einstein condensation occurs in momentum space". Explain the statement.

GROUP-A

Answer any *one* question from the following $10\times1 = 10$

2. (a) Show that if a generalised co-ordinate does not occur in the Lagrangian, the corresponding generalised momentum is conserved. 2

(b) Derive Lagrange's equations of motion for a holonomic conservative system. How will the result be modified for non-conservative system? 5+3

3. (a) Find the Hamiltonian and the Hamilton's equation of motion for a system whose Lagrangian is given by 4

$$L = \frac{1}{2}e^{\alpha t}\left(m\dot{q}^2 + kq^2\right)$$

(b) Show that the transformation given by $q = \sqrt{2p}\,\sin Q$ and $p = \sqrt{2p}\,\cos Q$ is canonical using Poisson Bracket. 3

(c) Define energy using the Lagrangian. Hence show that energy is constant in time if time does not explicitly appear in the Lagrangian. 3

GROUP-B

Answer any *one* question from the following 10×1 = 10

4. (a) A frame s' moves with uniform velocity 'u' along x' axis relative to a frame s. At time $t = t' = 0$, the origin of s and s' coincided and axes overlapped. In s frame, a projectile thrown with velocity 'v' describes a parabola given by $x = vt$, $y = \frac{1}{2}ft^2$. Find its trajectory in s' - frame. 4

(b) Obtain Einstein's formula for addition of velocities. 3

(c) Explain what is meant by space-like, time-like and light-like four-vectors. 3

5. (a) In one space and one time dimensions draw the following: (1+1)+(1+1)

(i) Two events A and B that are causally disconnected. Next, draw another point C, which is causally connected with both A and B.

(ii) The world lines for a particle at rest and that of a photon.

(b) (i) Define what is meant by proper time. 2+1+3

(ii) Define four-momentum p^α.

(iii) Using the definition of p^α show that $p^\alpha p_\alpha = m^2c^2$.

GROUP-C

Answer any *two* questions from the following 10×2 = 20

6. (a) What do you understand by micro-canonical, canonical and grand canonical ensemble? 3

(b) Let N_1, N_2 and N_3 be the number of particles with energies E_1, E_2 and E_3 and degeneracy g_1, g_2 and g_3 respectively, constitute a micro-canonical ensemble. Show that thermodynamic probability for the system is 3

$$\frac{(N_1 + N_2 + N_3)!}{N_1!\ N_2!\ N_3!}\, g_1^{N_1}\ g_2^{N_2}\ g_3^{N_3}$$

(c) A system consists of N distinguishable particles having spin $\pm 1/2$, in a magnetic field B can occupy any of its two energy states $\pm \mu B$. If at any moment n numbers of particles are in high energy state ($i.e.$ $+\mu B$), find the Energy and entropy of the system. 4

7. (a) What is Bose-Einstein statistics? What are the basic postulates used? 1+1+4

Derive an expression $n_i = \dfrac{g_i}{e^{\alpha - 1\beta E_i} - 1}$ for the most-probable distribution of the particles of a system obeying B.E. statistics and hence deduce Planck's black-body radiation formula.

(b) Define Fermi energy and Fermi temperature. Explain the significance of Fermi energy. 3

(c) According to which statistics, the energy at absolute zero cannot be zero? 1

8. (a) What is electron gas? Starting from Fermi-Dirac distribution law derive the expression for energy distribution of free electrons in a metal. 5

(b) Calculate the value of Fermi energy at absolute zero temperature. 2

(c) Three particles are to be distributed in four energy levels *a*, *b*, *c*, *d*. Calculate all possible ways of this distribution when particles are (i) Ferminons, (ii) Bosons, (iii) Classical particles. 3

UNIT-VB

9. Answer any ***five*** questions from the following: 2×5 = 10

(a) Explain the origin of fine structure splitting.

(b) Calculate the Lande's g factor for s-electron.

(c) What is de Broglie wavelength for electron moving with velocity $v = \frac{3}{5}c$?

(d) Show that the momentum operator $\hat{p}_x$ is Hermitian.

(e) How can you normalize the free-particle wave function?

(f) Write down the conditions of admissibility of the wave function.

(g) Why do molecules show band spectra rather than line spectra?

(h) A diatomic molecule does not rotate in ground state — Why?

GROUP-D

Answer any *three* questions from the following 10×3 = 30

10.(a) What is Compton effect? Do you observe Compton effect with visible light? Give reasons. 1+2

(b) Show that the change in wavelength of the photon in Compton effect is given by $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$, where the symbols have their usual meanings, ϕ the angle of scattering. 4

(c) Show that the energy of the recoil electron is given by $E_k = h\nu \frac{2\alpha\cos^2\theta}{(1+\alpha)^2 - \alpha^2\cos^2\theta}$, where $\alpha = \frac{h\nu}{m_0 c^2}$ and θ is the angle of recoil. 3

11.(a) State Heisenberg Uncertainty Principle. What is its importance? 2

(b) Illustrate the above principle with the help of electron diffraction experiment through single slit. 2

(c) Applying Heisenberg uncertainty principle prove that the lowest energy of the linear Harmonic oscillator is $\frac{1}{2}\hbar\omega$. 3

(d) Find the average time that the atomic system remains in energy state whose spectral line width is 10^{-4} Angstrom unit (Å) for $\lambda = 4000$ Å. 3

12.(a) Show that $[x, [x, \hat{H}]] = -\frac{\hbar^2}{m}$, where $\hat{H}$ is given by $\hat{H} = \frac{p^2}{2m} + v(x)$. 2

(b) Show that for a three dimensional wave packet 4

$$\frac{d}{dt}\langle x^2\rangle = \frac{1}{m}\left[\langle xp_x\rangle + \langle p_x x\rangle\right]$$

(c) Write down the Hamiltonian operator $\hat{H}$ for a linear Harmonic oscillator. Verify that the function $\psi(x) = \left(\frac{2\alpha}{\sqrt{\pi}}\right)^{1/2} \alpha x \, e^{-\alpha^2 x^2/2}$ is an eigenfunction of $\hat{H}$, where α is a constant. What is the corresponding eigenvalue? 1+2+1

13.(a) Find out the radial probability function for the ground state of hydrogen atom. 2

(b) Show that the most probable distance of the electron from the nucleus in the ground state of hydrogen atom is equal to Bohr's radius. 3

(c) Find the expectation value of Potential Energy of electron in hydrogen atom in the 1s state. The wave function for the electron in 1s state is given by 3+2

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

Also verify that this wave function is normalised.

GROUP-E

Answer any *one* question from the following 10×1 = 10

14.(a) (i) What is the origin of continuous X-ray spectra? What is meant by the shortest wave-length limit of continuous X-ray spectrum and how it is determined? In a conventional X-ray tube, an accelerating potential of 50 keV is used to accelerate the electrons. Find the shortest wave-length limit of the continuous X-ray spectrum produced. (1+1+1+1)+2

(ii) Explain the origin of characteristic X-ray spectrum.

(b) Discuss the goal of Stern-Garlach experiment. Why is it necessary to apply an inhomogeneous magnetic field in this experiment? 3+1

15.(a) (i) What is meant by space quantization? What role does magnetic quantum number play in space quantization? Explain in the light of vector atom model. 3+3

(ii) One very important quantum mechanical aspect of angular momentum $\vec{L}$ is incorporated in the vector atom model through the precession of $\vec{L}$ about a fixed direction (say, z-axis). Mention which aspect of $\vec{L}$ it is and explain how the precession actually takes it into account.

(b) What is normal Zeeman effect? Under what conditions it may be observed? 2

(c) Briefly explain why the intensities of Stokes' lines are greater than that of anti-Stokes' lines in Raman spectra. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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