WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER: MTMA-VIII-A

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

SECTION-I
(LINEAR ALGEBRA)

Answer any *one* question from the following $10\times1 = 10$

1. (a) If V and W are two finite dimensional vector spaces and $T: V \to W$ is a linear transformation, then show that 3

$$\dim V = \text{Nullity of } T + \text{Rank of } T$$

(b) A linear mapping $T:\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is defined by 3

$$T(x_1,\ x_2,\ x_3) = (2x_1 + x_2 - x_3,\ x_2 + 4x_3,\ x_1 - x_2 + 3x_3),\ (x_1, x_2, x_3) \in \mathbb{R}^3$$

Find the matrix of T relative to the ordered bases (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$.

(c) Let V be the vector space of all real polynomials of degree ≤ 3 and $T: V \to V$ be defined by $T(p(x)) = p'(x) + x^2 p''(x)$ (where $p'(x)$ and $p''(x)$ denote the first and second order derivatives of $p(x)$ respectively). 4

Show that T is a linear transformation. Find the matrix of T with respect to the ordered basis $\{1, x, x^2, x^3\}$.

2. (a) Let $T\colon V \to U$ and $S\colon U \to W$ be linear maps where, V, U, W are finite dimensional vector spaces over a field F. Then relative to a choice of ordered bases, show that $m(S\cdot T) = m(S)\cdot m(T)$ [where $m(T)$ stands for the matrix of T with respect to the chosen basis]. 5

(b) For a positive integer n, P_n denotes the vector space of polynomials of degree $\le n$, over the field of real numbers. Let $T: P_2 \to P_3$ be a linear transformation defined by 3

$$T\big(f(x)\big) = 2f'(x) + \int_0^x 3f(t)\,dt,\ \text{ for all } f(x) \in P_2$$

Prove that T is injective.

(c) The matrix representation of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ is $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ relative to the standard basis of $\mathbb{R}^3$. Find the explicit representation of T. 2

SECTION-II
(MODERN ALGEBRA)

Answer any *one* question from the following $8\times1 = 8$

3. (a) If H be a subgroup of a commutative group G then prove that the quotient group G/H is commutative. Is the converse true? — Justify with example. 3+2

   (b) In a group G, let H, K be subgroups such that K is normal in G. Prove that $KH = \{kh \,/\, k\in K\ ,\ h\in H\}$ is a subgroup of G. 3

4. (a) If $(H, \circ)$ is a normal subgroup of a group $(G, \circ)$, then prove that the quotient group $(G/H\,, *)$ is Abelian if and only if 4

$$x \circ y \circ x^{-1} \circ y^{-1} \in H \quad \forall x,\, y \in G$$

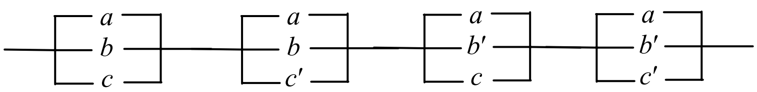
   (b) If G is a commutative group and $\phi : G \to G'$ is an epimorphism from G to any group G', then show that G' is also commutative. Is the converse true? Justify. 2+2

SECTION-III
(BOOLEAN ALGEBRA)

Answer any *one* question from the following $7\times1 = 7$

5. (a) In a Boolean algebra B, prove that $a \vee x = a \vee y$ and $a' \vee x = a' \vee y$ implies $x = y$. Write the dual of the statement. 3+1

   (b) Simplify the circuits. 3



6. (a) Express the Boolean expression $(x+y)(x+y')(x'+z)$ in DNF in the variable x, z and also express it in DNF in the variables x, y, z. 4

   (b) If x, y, z are three switches, then draw a switching circuit representing $[zx + x(y+z')]\,(z+x)\,(z+y)$. 3

GROUP-B
(DIFFERENTIAL EQUATION-III)

Answer any *one* question from the following $15\times1 = 15$

7. (a) Find the power series solution of $y'' + (x-3)y' + y = 0$ near $x = 2$. 5

(b) If $L\{F(t)\}=f(s)$, and $G(t)=\begin{cases}F(t-a) & t>a\\ 0 & t<a\end{cases}$, then prove $L\{G(t)\}=e^{-as}f(s)$. Where L denotes Laplace transform. 5

(c) Use the convolution theorem to find $L^{-1}\left\{\dfrac{s^2}{(s^2+4)^2}\right\}$, where L^{-1} denotes inverse Laplace transform. 5

8. (a) Applying power series method, solve $\dfrac{d^2y}{dx^2}-y=x$. 5

(b) If $F(s)=\dfrac{1}{(s^2+a^2)(s^2+b^2)}$ $(a\neq b)$, then find $f(t)$, where $f(t)=\mathrm{L}^{-1}\{F(s)\}$. 5

(c) Using first shifting property of Laplace transform evaluate 5

$$\mathrm{L}^{-1}\left(\frac{s-10}{s^2-4s+20}\right)$$

GROUP-C
(TENSOR CALCULUS)

Answer any *one* question from the following 10×1 = 10

9. (a) Prove that Kronecker delta δ^i_j is a mixed tensor of type (1, 1). 3

(b) Define the null vector. The line element in V_4-space is given by 1+3

$$ds^2=-(dx^1)^2-(dx^2)^2-(dx^3)^2+c^2(dx^4)^2$$

Verify whether $\left(-1,\, 0,\, 0,\, \dfrac{1}{c}\right)$ is the null vector in V_4.

(c) Prove that all Christoffel symbols are zero in the Euclidean space. 3

10.(a) Line element of two neighboring points $P(x^i)$ and $Q(x^i+dx^i)$ in a 3-dimensional space is given by 4

$$ds^2=(dx^1)^2+2(dx^2)^2+3(dx^3)^2-2dx^1dx^2+4dx^2dx^3$$

By this line element, does the above space form a Riemannian space? Justify it.

(b) Show that $g_{ik,j}=0$ and $\delta^i_{k,j}=0$. 2+1

(c) Show that $g_{ij}\,dx^i dx^j$ is an invariant, where g_{ij} is the fundamental metric tensor. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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