WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER: MTMA-VII

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

(VECTOR ANALYSIS-II)

Answer any *one* question from the following 10×1 = 10

1. (a) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^2}\, dS$ where S is any closed surface enclosing a volume V. 5

   (b) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path given by $x = t$, $y = t^2$, $z = t^3$. 5

2. (a) Prove that $\int_V \vec{\nabla}\phi\, dv = \int_s \phi\hat{n}\, ds$. In particular, show that $\int_s \hat{n}\, ds = \vec{0}$. 5

   (b) State Gauss divergence theorem. Use the theorem to show that $\iint_S \vec{r}\cdot d\vec{s} = 3V$, where V is the volume enclosed by the surface S and $\vec{r}$ has its usual meaning. 5

GROUP-B

(ANALYTICAL STATICS)

Answer any *five* questions from the following 7×5 = 35

3. Explain the terms 'force of friction' and angle of friction. A uniform ladder with its lower end on a rough ground leans against a smooth vertical wall. Prove that

   (i) If the inclination θ of the ladder to the wall is less than the angle of friction λ, no load placed on the ladder, however large, can make it slip;

   (ii) If $\lambda < \theta < \tan^{-1}(2\tan\lambda)$, the ladder can be made slip by placing on it an additional load, and

   (iii) If $\theta > \tan^{-1}(2\tan\lambda)$ the ladder will slip without any additional load.

4. Define Poinsot's central axis of a system of forces acting on a body and show that the central axis is unique.

5. A solid frustum of paraboloid of revolution of height h and latus rectum $4a$, rests with its vertex on the vertex of another paraboloid of revolution whose latus rectum is $4b$. Show the equilibrium is stable if $h < \frac{3ab}{a+b}$.

6. Find the centre of gravity of the area of the cardioid $r = a(1+\cos\theta)$.

7. Determine the conditions of equilibrium of a particle constrained to rest on a rough plane curve $y = \phi(x)$ under the action of any given forces.

8. What is the energy test of stability? Establish the energy test of stability for a rigid body with one degree of freedom only, in equilibrium under conservative forces.

9. A body rests in equilibrium on another fixed body having enough friction to prevent sliding, the portion of the two bodies in contact are spherical and of radii r and R respectively and the line joining their centers in position of equilibrium is vertical. Show that the equilibrium is stable if $\frac{1}{h} > \frac{1}{r} + \frac{1}{R}$ where h is the height of the C.G. of the body in position of equilibrium above the point of contact.

10. Two uniform similar rods of same material PQ and QT of lengths $2l$ and $2L$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angle α and β respectively to the vertical, prove that $(l^2 + 2lL)\sin\alpha = L^2 \sin\beta$.

11. Explain the astatic equilibrium of a system of coplanar forces acting at different points of a body and obtain the astatic centre.

GROUP-C

(RIGID DYNAMICS)

Answer any *two* questions from the following $15\times2 = 30$

12.(a) Find whether a given straight line is, at any point of its length, a principal axis of a material system. If it is so, then find the other two principal axes at that point. Hence show that if an axis passes through the centre of inertia of a body and is a principal axis at some point of its length, then it is a principal axis at all points of its length. 8

(b) Prove that the moment of inertia of a triangular lamina ABC about a perpendicular to the plane through the vertex A is $\frac{M}{3}(3b^2 + 3c^2 - a^2)$, where a, b, c are lengths of the sides of the triangle and M is its mass. 7

13.(a) State D'Alembert's principle and deduce the equation of motion of the centre of inertia of a rigid body and the equation of motion relative to the centre of inertia. 8

(b) A right cone of angle 2α, can turn freely about an axis passing through the centre of base and perpendicular to its axis. If the cone starts from rest with its axis horizontal, show that when the axis is vertical, the thrust on the fixed axis is to the weight of the cone as $\left(1+\frac{1}{2}\cos^2\alpha\right):\left(1-\frac{1}{3}\cos^2\alpha\right)$. 7

14.(a) Show that the resultant kinetic energy of a rigid body moving in two dimensions under finite forces is equal to the sum of two kinetic energies, one due to translation and the other due to rotation. 8

(b) A rough uniform rod, of length $2a$, is placed on a rough table at right angles to its edge. If the centre of gravity of the rod be initially at a distance b beyond the edge, then show that the rod will begin to slide when it has turned through an angle given by $\tan\theta=\frac{\mu a^2}{a^2+9b^2}$. 7

GROUP-D

(HYDROSTATICS)

Answer any *two* questions taking *one* question from each section.

SECTION-I

$15\times1 = 15$

15.(a) A mass of liquid is in equilibrium under the action of conservative system of forces. Show that the surface of equi-pressure, equi-density, and equi-potential energy coincide. If the system of forces is the force of gravity only, show that these surfaces are horizontal. 8

(b) A given volume V of a heavy liquid is acted on by forces $-\mu x$, $-\mu y$, $-\mu z$. Find the equation of the free surface. 7

16.(a) Prove that the depth of centre of pressure of a plane area immersed in a liquid under gravity is greater than that of the centre of mass of the area. What happens when the area is lowered further? 8

(b) If a floating solid be a cylinder, with its axis vertical, the ratio of whose specific gravity to that of the fluid is σ, prove that the equilibrium will be stable, if the radius of the base to the height is greater than $[2\sigma(1-\sigma)]^{\frac{1}{2}}$. 7

SECTION-II

$10\times1 = 10$

17.(a) A hemispherical surface of radius a is immersed in a liquid of density ρ with its centre at a depth h and its base inclined at an angle θ to the horizontal. Find the resultant thrust on the curved surface. 5

(b) A liquid fills the lower half of a circular tube of radius a in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is about to separate in two parts, show that $\omega = \sqrt{\frac{2g}{a}}$. 5

18.(a) A cone of density ρ whose height is h and the radius of whose base is a floating with its axis vertical and vertex upwards in liquid of density σ. Prove that the equilibrium is stable if $\rho < \sigma(1-\cos^6\alpha)$. 5

(b) Prove that if the temperature in the atmosphere falls uniformly with the height ascended, the height of a station above the sea level is given by $z = a\left\{1-\left(\frac{h}{h_0}\right)^m\right\}$, where h, h_0, are the reading of the barometer at station and sea level respectively and a, m are constants. 5

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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