WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER: MTMA-VI

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *two* questions from Question No. 1 to 3 and any *one* from Question No. 4 and 5

1. Answer any ***three*** questions from the following: 5×3 = 15

(a) Give the axiomatic definition of probability. Deduce the classical definition from the axioms. 2+1+2

Give the frequency interpretation of the axioms of probability.

(b) If a die is thrown k times, show that the probability of even number of sixes is $\frac{\{1+(2/3)^k\}}{2}$. 5

(c) From the numbers 1, 2, …… , (2n+1) three are chosen at random. Prove that the probability that these are in arithmetical progression is $\frac{3n}{(4n^2-1)}$. 5

(d) From an urn containing 3 white and 5 black balls, 4 balls are transferred into an empty urn. From this urn a ball is drawn and is found to be white. What is the probability that out of the four balls transferred 3 are white and 1 is black? 5

(e) If A_n be a monotone sequence of random events, then prove that 5

$$P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$$

2. Answer any ***three*** questions from the following: 5×3 = 15

(a) If a Random Variable $X \sim B(n, p)$, prove that $\mu_{k+1} = p(1-p)(nk\mu_{k-1} + \frac{d\mu_k}{dp})$, 5

where μ_k is the k-th central moment. Hence obtain the mean and variance of X.

(b) In the equation $x^2 + 2x - q = 0$, q is a random variable uniformly distributed over the interval (0, 2). Find the distribution function of the larger root. 5

(c) The joint density function of the random variables X, Y is given by 5

$$f(x, y) = K(3x + y) \text{ , when } 1 \le x \le 3 \text{ and } 0 \le y \le 2,$$

$$= 0 \quad \text{, otherwise}$$

Find: (i) $P(X + Y < 2)$ (ii) The marginal distribution of X and Y. Investigate whether X and Y are independent.

(d) X is a continuous random variable having a probability density function (p.d.f) $f_X(x)$. Let y be a continuously differentiable function of x. Show that the p.d.f $f_Y(y)$ of random variable $Y = g(X)$ is given by $f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$. 5

(e) If X follows standard normal distribution, show that $\frac{1}{2}X^2$ follows gamma distribution with parameter $\frac{1}{2}$. 5

3. Answer any ***three*** questions from the following: 5×3 = 15

(a) Show that the mean deviation about the mean of a normal (m, σ) distribution is $\sqrt{\frac{2}{\pi}}\,\sigma$. 5

(b) If X and Y are two correlated random variables with same standard deviation, show that the correlation coefficient between X and $X + Y$ is $\sqrt{\frac{(1+\rho)}{2}}$, where ρ is the correlation coefficient between X and Y. 5

(c) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant $k(0 < k < 1)$ is $k(1 - \log k)$. 5

(d) Find the moment generating function of a random variable X uniformly distributed in $(-a, a)$ and hence find $E(X^n)$, n a positive integer. 5

(e) A random variable X has a density function $f(x)$ given by 5

$$f(x) = e^{-x}, \quad x \geq 0$$
$$= 0 \quad , \text{ elsewhere}$$

Show that Tchebycheff's inequality gives $P(|X - 1| \geq 2) \leq \frac{1}{4}$. Show that the actual probability of this event is e^{-3}.

4. (a) Define an unbiased and consistent estimate of a parameter connected with the distribution function of a population. Prove that sample mean is always unbiased and consistent estimate of the population mean. 2+4

(b) Find the maximum likelihood estimate of the parameter α of the continuous population having the density function $f(x) = (1+\alpha)x^{\alpha}$, $0 < x < 1$ where $\alpha > -1$. 6

(c) What is meant by a statistical hypothesis? 2+6

A drug is given to 10 patients and the increment of blood pressure were recorded to be 3, 6, – 2, 4, – 3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure? It may be assumed that for 9 degrees of freedom $P(t > 2.262) = 0.025$.

5. (a) The bivariate probability density function of two random variables X and Y is given by 8

$$f(x, y) = x + y \quad , \quad 0 < x < 1,\ 0 < y < 1$$
$$= 0 \quad , \quad \text{elsewhere}$$

Calculate the means, standard deviations of X and Y and also the correlation coefficient between X and Y. Find the equations of two regression lines.

(b) For two random variables X and Y with the same mean, the two regression lines are $y = ax + b$ and $x = \alpha y + \beta$. Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Find also the common mean. 5

(c) The heights of 10 males of a normal population are found to be 70, 67, 62, 67, 61, 68, 70, 64, 65, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level, assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$. 7

GROUP-B

SECTION-I

[Marks: 30]

Answer any *three* questions from the following $10 \times 3 = 30$

6. (a) What are the different sources of computational errors in a numerical computational work? Discuss with suitable examples. 2

(b) Find the relative percentage error in $f(x)$ for $x = 0$, if the error in x is 0.002, where $f(x) = x^2 - 6x + \sin x$. 2

(c) Define a confluent divided difference of order one. 2

(d) Write down the remainder term associated with Newton's forward interpolation formula with $(n+1)$ equispaced interpolating points $x_0, x_1, \ldots, x_n$. Hence show that the maximum absolute error in linear interpolation is given by $h^2 M_2 / 8$ where M_2 is $\max_{x_0 \le x \le x_1} |f''(x)|$. 4

7. (a) Explain the Newton-Raphson method for computing a simple root of an equation $f(x) = 0$. When does the method fail? 4+1

(b) Obtain Lagrange's interpolation formula (without error term). 5

8. (a) State the general principle of Newton-Cotes' closed type formula for evaluating an integral of the form $\int_a^b f(x)\,dx$ where a, b are finite. Hence or otherwise obtain the trapezoidal rule. 4+1

(b) Describe Gauss' Elimination method for numerical solution of a system of linear equations and explain the pivoting process in this connection. 5

9. (a) Deduce numerical differentiation formula (both 1st and 2nd order) from Newton's forward interpolation formula. 5

(b) Using Picard's method, find a solution of $\frac{dy}{dx} = 1 + xy$ up to the third approximation, when $x_0 = 0,\ y_0 = 0$. 5

10.(a) Explain the method of Regula-Falsi for computing a real root of an equation $f(x) = 0$ and also explain the geometrical interpretation of the process. 5

(b) Solve the equation $\frac{dy}{dx} = x^2 + y^2,\ \ y(0) = 1$ by fourth order Runge-Kutta method for $x = \mathrm{O}(0.1)\ 0.2$ correct up to 4D. 5

SECTION-II

[Marks: 20]

Answer any *two* questions from the following 10×2 = 20

11.(a) Discuss primary memory and secondary memory. What is the fundamental unit of measuring memory? 3+2

(b) Draw a flowchart to find *n*! (*n* is a positive integer).

(c) (i) Convert $(520.375)_{10}$ into octal form. 2+2+1

(ii) Use 2's complement to compute $(1110.1001)_2 - (1010.011)_2$.

(iii) Find the CNF of $xy + x'y$.

12.(a) Write a FORTRAN 77/90 or C program to input 5 numbers and print the biggest of the five. 5

(b) Write a FORTRAN 77/90 or C program to evaluate 5

$$e = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots\cdots + \frac{1}{100} + \cdots\cdots \text{ correct to 4D}$$

13.(a) Write the full forms of ALU, ROM, CPU, CNF. What is a flowchart? 2+1

(b) Draw a flowchart to determine the smallest of three numbers. 2

(c) Write a FORTRAN or C program to find the sum of two $m \times n$ matrices. 5

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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