WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2021-22

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Does $\lim\limits_{(x,\, y)\to(0,\, 0)} \dfrac{2xy^3}{x^2+y^6}$ exist? Give reasons. 2

(b) Use ε-δ definition of the limit to prove $\lim\limits_{x\to -3} x^2 = 9$. 2

(c) Find the coordinates of the points on the curve $y = x^3 - 6x + 7$ where the tangent is parallel to $y = 6x + 1$. 2

(d) Find domain of the function $f(x) = \sqrt{x-1} + \sqrt{5-x}$. 2

(e) Is Rolle's theorem applicable for the function $f(x) = x^2 - 5x + 6$ in [1, 4]? Justify your answer. 2

(f) Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$. 2

(g) Prove that the function $f:\mathbb{R}\longrightarrow\mathbb{R}$ defined by $f(x) = \sin x$, $x\in\mathbb{R}$ is continuous on $\mathbb{R}$ by using the ε-δ definition of continuity. 2

(h) Examine the nature of discontinuity of the function f defined by 2

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}} & x > 0 \\ 0 & x = 0 \end{cases}$$

at 0.

(i) Find the curvature of the parabola $x^2 = 12y$ at the point $\left(-3, \frac{3}{4}\right)$. 2

2. (a) A function f in [0, 1] is defined as follows 5

$$\begin{aligned} f(x) &= x^2 + x \quad , \quad 0 \le x < 1 \\ &= 2 \quad , \quad x = 1 \\ &= 2x^3 - x + 1 \quad , \quad 1 < x \le 2 \end{aligned}$$

Examine the differentiability of f at $x = 1$. Is f continuous at $x = 1$?

(b) If $f : I \longrightarrow \mathbb{R}$ is a function differentiable at a point $c \in I$, then show that it is continuous at c. 3

3. (a) If $x = \sec\theta - \cos\theta,\ y = \sec^n\theta - \cos^n\theta$, show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$. 4

(b) If $\lim\limits_{x\to 0} \frac{a\sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a and the limit. 4

4. (a) If $f(x) = \sin x$, find the limiting value of θ, when $h \to 0$ using the Lagrange's mean value theorem $f(a+h) = f(a) + h f'(a+\theta h)$, $0 < \theta < 1$. 4

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{4}{x+y+z}$. 4

5. (a) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$, prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. 4

(b) If $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$. 4

6. (a) Find radius of curvature of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$ at any point θ. 4

(b) Find the asymptotes of the equation $(a+x)^2(b^2+x^2) = x^2y^2$. 4

7. (a) Expand e^x in ascending powers of $(x-1)$. 4

(b) Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6$ in [1, 3]. 4

8. (a) Prove that $\frac{2x}{\pi} < \sin x < x$ for $x > 0$. 4

(b) Find the greatest and the least value of $2\sin x + \sin 2x$ in the interval $\left(0, \frac{3\pi}{2}\right)$. 2+2

9. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally. 4

(b) Find the points on the parabola $y^2 = 2x$ which is nearest to the point (3, 0). 4

10.(a) Find the values of a and b such that the function 5

$$
\begin{aligned}
f(x) &= x+\sqrt{2}a\sin x \quad , \quad 0 \le x \le \frac{\pi}{4} \\
&= 2x\cot x + b \quad , \quad \frac{\pi}{4} < x \le \frac{\pi}{2} \\
&= a\cos 2x - b\sin x \quad , \quad \frac{\pi}{2} < x \le \pi
\end{aligned}
$$

is continuous for all values of x in the interval $0 \le x \le \pi$.

(b) If $u(x,\ y) = \cot^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\sin 2u = 0$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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