WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

STSACOR02T-STATISTICS (CC2)

MATHEMATICAL ANALYSIS AND ALGEBRA-I

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions — 5×4 = 20

1. Define subsequence of a real sequence. Show that if a monotone sequence has a convergent subsequence then the sequence is convergent. — 1+4

2. Distinguish between maximum and sepremum of a set $A(\subset\mathbb{R})$. Find the Supremum and Infimum of the set $A=\lim\limits_{n\to\infty}A_n$ with $A_n=\left(2-\frac{1}{n},\,5+\frac{1}{n^2}\right]$ $,n\geq 1$. — 2+3

3. Check whether the following series are convergent: — 5

   (i) $\sum\limits_{n\geq 1}\frac{n!}{r^n}$ with $r>0$

   (ii) $\sum\limits_{n\geq 1}\frac{1}{2^n}\left(1+\frac{1}{n}\right)^n$

   (iii) $\sum\limits_{n\geq 1}\sin\left(\frac{\pi}{n^2}\right)$.

4. Let S be a subspace of the Euclidean vector space $\mathbb{E}^4$ and let S be spanned by the vector $(0,-1/\sqrt{5}\,,0,2/\sqrt{5}\,)$. Extend the basis of S to obtain an orthonormal basis of $\mathbb{E}^4$. — 5

5. Distinguish between spanning set and set of basis vectors with an example. Consider the vectors $x_1=(1,3,2)$ and $x_2=(-2,4,3)$ in $\mathbb{E}^3$. Show that the set spanned by x_1 and x_2 is given by — 2+3

$$\{(\xi_1,\xi_2,\xi_3):\xi_1-7\xi_2+10\xi_3=0\}.$$

6. What is a non-singular matrix? Define adjoint of a matrix. For a square matrix A of order n, show that $|\operatorname{Adj}(A)| = |A|^{n-1}$. 1+1+3

GROUP-B

Answer any *three* questions 10×3 = 30

7. (a) Define Cauchy sequence of real numbers. State its relation with convergent sequence. Show that the sequence $a_n = \dfrac{\sin(n)}{\sqrt{n}}$, $n \geq 1$ is a Cauchy sequence. 2+1+2

(b) Let $\{a_n\}$ be a sequence of positive real numbers such that for $n \geq 1$, $a_{n+1} \leq a_n + \dfrac{1}{(n+1)^2}$. Show that the sequence is convergent. 5

8. (a) Define limit superior ($\overline{a}$) and limit inferior ($\underline{a}$) of a sequence $\{a_n\}$ of real numbers. Find $\overline{a}$ and $\underline{a}$ for the sequence $a_n = \{(-1)^n + 1\}n^2$, $n \geq 1$. 2+4

(b) For two real series $\sum\limits_{n\geq 1} a_n$ and $\sum\limits_{n\geq 1} b_n$ if $\sum\limits_{n\geq 1} a_n^2 < \infty$ and $\sum\limits_{n\geq 1} b_n^2 < \infty$ then show that $\sum\limits_{n\geq 1} a_n\, b_n$ converge absolutely. 4

9. (a) When is a set of vectors called linearly independent? Show that a set containing null vector cannot be linearly independent. 2+2

(b) Show that any vector in a vector space has a unique representation in terms of its basis. If V_1 and V_2 are two subspaces of a vector space, say V, then show that $V_1 \cap V_2$ is also a subspace of V, but $V_1 \cup V_2$ may not be the same. 2+4

10. (a) Show that the number of independent row vectors of a matrix is equal to the number of independent column vectors. 3

(b) Let X be any $m \times m$ matrix partitioned as $X = \left(X_1^{m\times r};\, X_2^{m\times(m-r)}\right)$ such that $X_2 = X_1 C$, where C is a $r \times (m-r)$ matrix. If the rows of X are linearly independent, show that the rows of X_1 are also linearly independent. 4

(c) If X be a $m \times n$ matrix having m linearly independent row vectors and $Y = XA$, where 3

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1\overline{n-1}} & a_{1n} \\ 0 & a_{22} & \cdots & a_{2\overline{n-1}} & a_{2n} \\ \vdots & \vdots & & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & & 0 & a_{nn} \end{pmatrix}$$

with $a_{ii} \neq 0\ \forall\ i = 1, 2, \ldots, n$. Prove that the rows of Y are linearly independent.

11. (a) State some basic properties of a polynomial $P_n(x)$ of degree n. 3

(b) What do you mean by equivalent polynomials? Describe its utility in finding roots. 1+2

(c) Write how you represent a polynomial of the form 4

$$P_n(x) = x^n + c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_1 x + c_0$$

in terms of its roots.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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