WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If a, b, c are all positive and $abc = k^3$, then prove that $(1+a)(1+b)(1+c) \geq (1+k)^3$.

(b) Solve the equation $3z^5 + 2 = 0$.

(c) Apply Descartes' rule of sign to determine the number of positive, negative and complex roots of the equation $x^5 - x^4 - 2x^2 + 2x + 1 = 0$.

(d) Prove that $2^{3n} - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

(e) If $\gcd(a, b) = 1$, then show that $b \mid ap \Rightarrow b \mid p$.

(f) Find a map $f : \mathbb{N} \longrightarrow \mathbb{N}$ which is one to one but not onto.

(g) Let $f : A \to B$ be an onto mapping and P, Q be subsets of B. Prove that $f^{-1}(P \cap Q) = f^{-1}(P) \cap f^{-1}(Q)$.

(h) Find the minimum number of non-real roots of the polynomial equation $x^8 + x^4 - x^2 = 0$.

(i) Give an example of a reflexive and symmetric relation on the set {1, 2, 3} which fails to be an equivalence relation on {1, 2, 3}.

2. (a) If a_1, a_2, a_3, a_4 be distinct positive real numbers and $s = a_1 + a_2 + a_3 + a_4$, then show that $\dfrac{s}{s-a_1} + \dfrac{s}{s-a_2} + \dfrac{s}{s-a_3} + \dfrac{s}{s-a_4} > 5\frac{1}{3}$. 3

(b) Show that $(n+1)^n > 2^n n!$. 2

(c) If A be the area and $2s$ the sum of the three sides of a triangle, show that $A \leq \dfrac{s^2}{3\sqrt{3}}$. 3

3. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that 4

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$$

(b) If z_1, z_2 are complex numbers such that z_1+z_2 and $z_1 \cdot z_2$ are both real then show that either z_1, z_2 are both real or $z_1 = \bar{z}_2$. 4

4. (a) Solve the equation $x^3-3x-1=0$, by Cardan's method. 4

(b) Form a biquadratic equation with rational coefficients, two of whose roots are $\sqrt{3}\pm 2$. 4

5. (a) Let X be any non-empty set. Prove that there does not exist any surjective map from X to $P(X)$, the power set of X. 2

(b) Prove that the relation ρ on $\mathbb{R}$ defined by $x\rho y$ if and only if $x-y \in \mathbb{Q}$ $(x,\ y \in \mathbb{R})$ is an equivalence relation. Find the equivalence class containing the element 0. 2+1

(c) A relation ρ on $\mathbb{R}$ is defined as follows: 3

$$a\rho b \text{ if and only if } |a| \le b$$

Show that ρ is transitive but neither reflexive nor symmetric.

6. (a) If p is a prime greater than 3, then show that $2p+1$ and $4p+1$ can not be primes simultaneously. 2

(b) Use mathematical induction to prove that for any positive integer n 3

$$1.2+2.2^2+3.2^3+\dots\dots.+n.2^n=(n-1)2^{n+1}+2$$

(c) Prove that for any positive integer n, $3^{4n+2}+5^{2n+1} \equiv 0 \pmod{14}$. 3

7. Transform the matrix $A=\begin{pmatrix} 1 & 2 & -1 & 10 \\ -1 & 1 & 2 & 2 \\ 2 & 1 & -3 & 2 \end{pmatrix}$ to its row reduced echelon form. 4+2+2=8

Hence find $\operatorname{rank} A$ and the solution set of the system of linear equations given by

$$x+2y-z=10$$

$$-x+y+2z=2$$

$$2x+y-3z=2$$

8. (a) Use Cayley-Hamilton theorem to express A^{-1} as a polynomial in A and then compute A^{-1} where $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. 2+2

(b) Show that the eigen values of an orthogonal matrix are of unit modulus. 4

9. (a) If A be a square matrix, then show that the product of the characteristic roots of A is $\det A$. 3

(b) Find all the eigen values of the following real matrix: 2+3

$$A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Find one eigen vector corresponding to the largest eigen value found above.

10.(a) Express the matrix 3+2

$$A=\begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$$

as product of elementary matrices and hence, find A^{-1}.

(b) If $A=\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$, show that A^2 cannot have imaginary characteristic roots. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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