# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

## PHSACOR01T-PHYSICS (CC1)

### MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest**

1. Answer any ***ten*** questions from the following: 2×10 =20

(a) Sketch: $f(\theta)=\sin\theta\cos\theta$ for $0\le\theta\le 4\pi$.

(b) If $\vec{r}$ be the position vector of a point on a closed contour, prove that the line integral $\oint \vec{r}\cdot d\vec{r}=0$.

(c) If $\vec{A}$ and $\vec{B}$ are irrotational then prove that $\vec{A}\times\vec{B}$ is solenoidal.

(d) Find the Taylor series of the function $f(x)=\dfrac{1}{x^2+4}$ about the point $x = 0$.

(e) State the Uniqueness theorem of the solution of a differential equation for initial value problems.

(f) Find the volume of the parallelepiped whose edges are represented by $\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$.

(g) The position vector of a particle is $\vec{r}(t)=\cos(\omega t)\,\hat{i}+\sin(\omega t)\,\hat{j}$, where $\omega$ is a constant. Prove that, at every instant, its velocity and acceleration are perpendicular to each other.

(h) Find the directional derivative of the scalar field $\phi(x,y,z)=2x^2+yz$ in the direction of the vector $\hat{i}+\hat{j}$ at the point (0, 1, –1).

(i) Find the value of $k$ so that the average value of the function

$f(t)=\dfrac{3t^2}{8}+kt$ in the range $0\le t\le 2$ vanishes.

(j) A particle moves along a curve, $x=2t^2$, $y=t^2-4t$ and $z=3t-5$ where "$t$" is time. Find its component velocity at time $t=1$ in the direction of vector $(\hat{i}-2\hat{j}+2\hat{k})$.

(k) Solve the following differential equation.

$$ye^{y}dx=(y^3+2xe^{y})\,dy.$$

(l) Show that, $\vec{\nabla}\times\left(\dfrac{\vec{r}}{r^2}\right)=0$.

(m) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.

(n) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

2. (a) Evaluate $\iint_R x(y-1)\,dxdy$, where R is the region bounded by the parabola $y=1-x^2$ and $y=0$. — 3+3+4

(b) Prove that for a scalar field $\phi$,

$$\oint_S \phi\, d\vec{S} = \int_V (\vec{\nabla}\phi)\, dV \,,$$

where V is the volume bounded by the closed surface S.

(c) Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2\sin(4x)$.

3. (a) State Green's theorem in a plane. — 1+5+(1+3)

(b) Verify Gauss' divergence theorem for the vector field $\vec{F} = 4y\hat{i} - 2x\hat{j} + z^2\hat{k}$, where $V$ is the volume bounded by the upper half surface of the sphere $x^2+y^2+z^2=1$ together with the plane $z=0$.

(c) State the condition of convergence of a Taylor series expansion. Find interval of $x$ for which the Taylor series of $\ln(1+x)$ about $x=0$, converges.

4. (a) If $r=\sqrt{x^2+y^2}$ and $z=\phi(r)$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{r}\phi'(r) + \phi''(r)$. — (2+2)+3+3

(b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

(c) If $\vec{A} = (y-2x)\hat{i} + (3x+2y)\hat{j}$, compute the circulation of $\vec{A}$ about a circle $C$ in the XY plane with center at the origin and radius 2, if $C$ is traversed in the positive direction.

5. (a) Find the area of the ellipse described by $x = a\cos\theta$, $y = b\sin\theta$. — 3+3+1+3

(b) Prove $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right)=0$, where $\vec{r}(\neq 0)$ is the position vector.

(c) What is meant by probability distribution function?

(d) A box ***A*** contains 2 white and 4 black balls. Another box ***B*** contains 5 white and 7 black balls. A ball is transferred from the box ***A*** to the box ***B***. Then a ball is drawn from the box ***B***. Find the probability that the drawn ball is white.

**N.B. :** *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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