WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

STSADSE02T-STATISTICS (DSE1/2)

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following — $5\times4=20$

1. Let $\{X_n\}$ be a sequence of *iid* random variables following $R(0,\theta),\ \theta>0$. Find the values $\alpha(>0)$ such that $n^{\alpha}(X_{(n)}-\theta)\xrightarrow{P}0$ as $n\to\infty$. Also check whether $X_{(n)}-X_{(m)}\xrightarrow{P}0$ as $n\to\infty$, where $m=\left[\frac{n+1}{2}\right]$. — 3+2

2. (a) Define consistent estimator. — 1+4

   (b) Suppose $\{(X_n, Y_n)'\}$ be a sequence of *iid* random vectors following $\mathcal{BN}$ (0, 0, 1, 1, ρ) with $|\rho|<1$. Define for $k\geq 1$,

$$U_k=\begin{cases}1 & \text{if } X_{2k-1}Y_{2k-1}+X_{2k}Y_{2k}>0\\ 0 & \text{if } X_{2k-1}Y_{2k-1}+X_{2k}Y_{2k}\leq 0\end{cases}$$

   Show that $T_n=2\overline{U}_n-1$ is a consistent estimator of ρ, where $\overline{U}_n=\frac{1}{n}\sum_{k=1}^{n}U_k$.

3. (a) State Lindeberg-Levy Central Limit Theorem. — 1+4

   (b) Let $\{X_n\}$ be a sequence of *iid* random variables with $E(X_1)=\mu$ and $Var(X_1)=\sigma^2\in(0,\infty)$. Find two real sequences $\{a_n\}$ and $\{b_n\}$ such that $b_n(\overline{X}_n^3-a_n)$ converges in distribution to some non-degenerating random variable *X*.

4. Let $\{X_n\}$ be a sequence of *iid* random variables following shifted exponential distribution with pdf: — 5

$$f(x)=\frac{1}{\sigma}\exp\left\{-\frac{(x-\mu)}{\sigma}\right\}I(x>\mu),$$

   where $I(\cdot)$ is the indicator function. Show that $Y_n=\dfrac{n(n-1)(X_{(1)}-\mu)}{\sum_{k=1}^{n}(X_k-X_{(1)})}\xrightarrow{\mathcal{D}}Y$ as $n\to\infty$, where *Y* follows exponential distribution with mean unity. [State the result(s) you need to use.]

5. State and prove the De-Moivre Laplace central limit theorem. 1+4

6. Define convergence in distribution for a sequence of random variables. Provide two examples (with justifications) where (i) convergence in distribution implies convergence in probability and (ii) convergence in distribution does not imply convergence in probability. $2+1\frac{1}{2}+1\frac{1}{2}$

GROUP-B

Answer any *two* questions from the following 10×2 = 20

7. (a) State and prove Delta method. 4+(3+3)

(b) Let $\{X_n\}$ be a sequence of *iid* random variables having finite forth moment. Show that

$$\sqrt{n}\begin{pmatrix}\overline{X}_n-\mu \\ m_2-\mu_2\end{pmatrix} \xrightarrow{\mathcal{D}} \mathcal{N}_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\mu_2 & \mu_3\\ \mu_3 & \mu_4-\mu_2^2\end{pmatrix}\right) \text{ as } n\to\infty,$$

where $\mathcal{N}_2$ stands for 2-dimensional normal (or bivariate normal) distribution. Also derive the asymptotic distribution of $\zeta_n = \overline{X}_n^2 / m_2$. Notations having their usual meaning.

8. (a) Write a note on Fisher's z-transformation. Also discuss Anscombe's correction for the transformation. (4+2)+4

(b) Suppose five famous football manufacturing companies produce footballs of different categories viz., football for kids, football for beginners, football for adults and football for professional players. The average annual sales of different types of footballs are provided for each of these companies. Justify, based on the given information, whether there is any preference among the companies on purchasing a specific type of football.

9. (a) State Weak Law of Large Numbers (WLLN) for a sequence of random variables. Derive a sufficient condition under which a sequence of independent random variables obeys WLLN. (2+2)+2+4

(b) Let $\{X_n\}$ a sequence of uniformly bounded random variables with negative covariances. Check whether WLLN holds for the sequence.

(c) Let $Y_k = \alpha + \beta x_k + e_k$, where x_k's are non-stochastic and $e_k \sim iid$ $(\text{mean}=0, \text{variance}=\sigma^2)$, $k \geq 1$.

Show that, $\hat{\beta}$, the last square estimator, is a CAN estimator of β provided

$$\gamma_n^2 = \frac{\max\limits_{1\leq k\leq n} (x_k-\bar{x})^2}{\sum_{k=1}^{n}(x_k-\bar{x})^2} \to 0 \text{ as } n\to\infty.$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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