WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

PHSADSE02T-PHYSICS (DSE1/2)

ADVANCED DYNAMICS

Time Allotted: 2 Hours　　　　Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***fifteen*** questions from the following: 2×15 =30

(a) Show that for a cyclic co-ordinate, the conjugate momentum is conserved.

(b) What do you mean by generalized co-ordinates and what is the advantage of using them?

(c) State whether the constraints given by $x\frac{dy}{dt} - y\frac{dx}{dt} = c$ (constant) is holonomic one.

(d) Define Poisson Bracket (PB) and write down the Hamilton's equation of motion using PB.

(e) How many degrees of freedom does a rigid body have when the body is rotating about an axis that is fixed in space?

(f) Show that if the Lagrangian of a system does not depend on time explicitly, its Hamiltonian is a constant of motion.

(g) State the parallel axes theorem of moment of inertia.

(h) What are principal moments of inertia and principal axes?

(i) Show that the directions of angular velocity and angular momentum, though usually differ, coincide only along principal axes.

(j) State the properties of principal moments of inertia of rigid body.

(k) What do you understand by stable and unstable equilibria?

(l) Find the eigen-frequencies of a vibrating system characterized by a Lagrangian

$$L = \frac{1}{2}(\eta_1^2 + \eta_2^2 + \eta_3^2) - \alpha^2(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_1\eta_2).$$

(m) A particle of mass m in one dimensional motion along x-axis $(x > 0)$ with its potential energy given by $V(x) = x + \frac{1}{x}$, is executing small oscillation near its stable equilibrium. Find the frequency of the oscillation.

(n) Potential energy of a particle is given by $V = x^4 - 4x^3 - 8x^2 + 48x$. Find the points of stable and unstable equilibria.

(o) What are 'Autonomous' and 'Nonautonomous' systems? Give example for each.

(p) Find and classify all the fixed points of the following first order differential equation:

$$\frac{dy}{dt} = X(y) = -y(y^2 - 4)$$

(q) Draw the 2D phase space diagram of a point particle of mass m falling freely under the action of earth's gravity.

2. (a) A pendulum bob of mass m is suspended by a string of length l from a point of support. The point of support moves along a horizontal x-axis according to the equation $x = a\cos\omega t$. Assuming the pendulum swings only in the vertical plane containing the x-axis. 3+1

(i) Set up the Lagrangian and write out the Lagrange equation.

(ii) Show that small values of the angle which the string makes with a line vertically downward, the equation reduces to that of a forced harmonic oscillator.

(b) Show that the gauge transformation $\mathbf{A}' = \mathbf{A} + \nabla f(\mathbf{r}, t)$, $\phi' = \phi - \frac{\partial f}{\partial t}$ effected by a generating function $F_2(\mathbf{r}, \mathbf{p}) = \mathbf{r}.\mathbf{p} - ef(\mathbf{r}, t)$ can be regarded as a canonical transformation. 3

(c) Sketch the phase portrait corresponding to $\dot{x} = x - \cos x$, and determine the stability of all the fixed points. 3

3. (a) The equation of a damped harmonic oscillator is $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$. Discuss the phase trajectory for $b^2 < \omega_0^2$ and $b^2 > \omega_0^2$. 4

(b) The Van der Pol's equation is given by $\frac{d^2x}{dt^2} - \varepsilon(1 - x^2)\frac{dx}{dt} + x = 0$. Write parametric equations of the system. 3

(c) What do you understand by a limit cycle? What is an attractor? $1\frac{1}{2}+1\frac{1}{2}$

4. (a) For rotational motion of rigid bodies, derive an expression for kinetic energy in terms of moment of inertia and angular velocity. 3

(b) Write down Euler's equation for free symmetrical top and solve for angular velocity ω. Show that the angular velocity vector ω rotates about the body symmetry axis describing a cone with the vertex at the origin. 3+1

(c) Show that the transformation $P = \frac{1}{2}(p^2 + q^2)$ and $Q = \tan^{-1}\frac{q}{p}$ is canonical. 3

5. (a) Find the canonical transformation generated by 2+1+1+2

$$F_1(Q, q) = \lambda q^2 \cot Q,$$

λ being a constant. If the Hamiltonian in (q, p) representation is

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2,$$

find the Hamiltonian in (Q, P) representation. Choose λ to make this Hamiltonian independent of Q and hence find the equation of motion in each representation.

(b) Draw the potential for the system $\frac{dx}{dt} = x - x^3$ and identify all the equilibrium points. 2

(c) Show that for a first order dynamical system (with, $\frac{dx}{dt} = f(x)$), there are no periodic solutions. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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