WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

PHSADSE01T-PHYSICS (DSE1/2)

ADVANCED MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Prove that the contraction of tensors A^p_q is an invariant.

(b) If $\varphi = a_{jk} A^j A^k$ then show that one can write $\varphi = b_{jk} A^j A^k$, where b_{jk} is symmetric.

(c) Show that the Kronecker delta is a mixed tensor of order two.

(d) For an orthogonal basis prove that norm of a nonzero vector is positive definite.

(e) Why is Laplace transformation a linear operation?

(f) Show that the Laplace transform of the integral of $f(x)$, i.e. $L\left[\int_0^x f(x)\,dx\right] = \frac{1}{p}\bar{f}(p)$, where $L[f(x)] = \bar{f}(p)$.

(g) Prove that every vector in a finite-dimensional vector space V over the field F can be uniquely expressed as a linear combination of the vectors of its basis.

(h) Show that the vectors (2, –5, 3) cannot be expressed as a linear combination of the vectors $\alpha_1 = (1, -3, 2)$, $\alpha_2 = (2, -4, -1)$ and $\alpha_3 = (1, -5, 7)$.

(i) Show that the four matrices $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ from a group under matrix multiplication.

(j) Find Laplace transform of $f(x)$, where $f(x) = \begin{cases} 0 \ , & x < a \\ 1 \ , & x > a \end{cases}$.

(k) Find Laplace transform of $e^{4x}\sin(2x)\cos(x)$.

(l) Examine if the following operator is linear:

$$Af(x) = x^2 f(x)$$

(m) Evaluate $L^{-1}\left[\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right]$.

2. (a) Show that the velocity at any point of a fluid $\dfrac{dx^k}{dt}=v^k$ is a tensor but acceleration $\dfrac{dv^k}{dt}$ is not a tensor. 2+1

(b) Using Levi-Civita symbol establish the relation $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{B})-\vec{C}(\vec{A}\cdot\vec{B})$. 4

(c) What are stress and strain tensors? Write down the tensorial form of Hooke's law in Elasticity. 3

3. (a) Distinguish between isomorphism and homomorphism in connection with two groups. 2

(b) Define basis and dimension of a vector space. 2

(c) Define orthogonal and orthonormal set of vectors. 2

(d) Show that the sets $S\{(1,\ 0,\ 0),\ (1,\ 1,\ 1),\ (0,\ 1,\ 0)\}$ spans a vector space R^3 but is not a basis set. 4

4. (a) For the set of basis vectors, (0, 2, 0, 0), (3, -4, 0, 0) and (1, 2, 3, 4) use Gram-Schmidt process to construct an orthonormal set. 4

(b) A linear transformation T is defined as $T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}x_1+x_2\\x_2-x_3\end{pmatrix}$ that transforms a vector a 3-D real space to 2-D real space. Show that the transformation matrix is $T=\begin{pmatrix}1&1&0\\0&1&-1\end{pmatrix}$. 2

(c) Use the convolution theorem to find the inverse Laplace transform of $\dfrac{1}{(s^2+4)^2}$. 4

5. (a) If $LT[f(x)]=\bar{f}(s)$, then prove that $LT[f''(x)]=s^2\bar{f}(s)-sf(0)-f'(0)$. 3

(b) Find inverse Laplace transform of $\dfrac{s+2}{s^2(s+1)(s-2)}$. 4

(c) Using Laplace transform solve the differential equation 3

$$2\frac{d^2y}{dx^2}+5\frac{dy}{dx}+2y=e^{-2x} \text{ with } y(0)=1,\ \frac{dy}{dx}(0)=1$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——