WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Why we introduce artificial variable in Charne's penalty method?

(b) Define extreme point of a convex set. Give an example of a convex set having no extreme point.

(c) Find in which half space of the hyperplane $2x_1+3x_2+4x_3-x_4=6$, the points. $(4,-3,2,1)$ and $(1,2,-3,1)$ lie.

(d) Prove that the solution of the transportation problem is never unbounded.

(e) Solve the following 2×2 game problem by algebraic method:

Player B

$$\text{Player A}\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}$$

(f) Find graphically the feasible space, if any for the following

$$2x_1+x_2\le 6$$

$$5x_1+3x_2\ge 15,\; x_1, x_2\ge 0$$

(g) Prove that if the dual problem has no feasible solution and the primal problem has a feasible solution, then the primal objective function is unbounded.

(h) Find the optimal strategies and game value of the following game problem.

Player B

Player A		
	9	5
	7	11

(i) Suppose you have a linear programming problem with five constraints and three variables. Then what problem, primal or dual will you select to solve? Give reasons.

2. (a) Solve graphically the L.P.P. 4

Maximize $z = 5x_1 - 2x_2$

Subject to $5x_1 + 6x_2 \geq 30$

$9x_1 - 2x_2 = 72$

$x_2 \leq 9$

$x_1, x_2 \geq 0$

(b) Show that the L.P.P. 4

Maximize $z = 4x_1 + 14x_2$

Subject to $2x_1 + 7x_2 \leq 21$

$7x_1 + 2x_2 \leq 21$

$x_1\ x_2 \geq 0$

admits of an infinite number of solutions.

3. Use Charne's Big-M method to solve the L.P.P. 8

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$, $x_1, x_2 \geq 0$

4. (a) Let x be any feasible solution to the primal problem and v be any feasible solution to its dual problem then prove that $cx \leq b^T v$. 4

(b) Find the dual of the following problem 4

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \leq 3$

$3x_2 - x_3 \geq 5$

$x_1, x_2 \geq 0, x_3$ is unrestricted in sign.

5. (a) Prove that a subset of the columns of the coefficient matrix of a transportation problem are linearly dependent if the corresponding cells or a subset of them can be sequenced to form a loop. 4

(b) Using North-West corner rule find the initial basic feasible solution of the following transportation problem hence find the optimal solution. 4

	D_1	D_2	D_3	D_4	a_j
O_1	2	1	3	4	30
O_2	3	2	1	4	50
O_3	5	2	3	8	20
b_j	20	40	30	10	

6. (a) Prove that the dual of the dual is the primal. 3

(b) Find the optimal assignment and minimum cost for the assignment problem with the following cost matrix: 5

	V	W	X	Y	Z
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

7. (a) In a two persons zero sum game, if the 2×2 pay-off matrix has no saddle point then find the game value and optimal mixed strategies for the two players. 4

(b) Solve graphically the following game problem: 4

	B_1	B_2	B_3	B_4
A_1	1	2	6	12
A_2	8	6	3	2

8. (a) Show that every finite two person zero sum game can be expressed as a linear programming problem. 4

(b) Solve the following game problem by converting it into a L.P.P.: 4

		Player Q		
		Q_1	Q_2	Q_3
	P_1	4	2	5
Player P	P_2	2	5	1
	P_3	5	1	6

9. (a) In a rectangular game, the pay-off matrix A is given by 6

$$A=\begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}$$

state, giving reason whether the players will use pure or mixed strategies. What is the value of the game?

(b) Let $(a_{ij})_{m\times n}$ be the pay-off matrix for a two person zero-sum game. Then prove that 2

$$\max_{1\le i\le m}\left[\min_{1\le j\le n}\{a_{ij}\}\right]\le \min_{1\le j\le n}\left[\max_{1\le i\le m,}\{a_{ij}\}\right]$$

10.(a) Solve the following game by graphical method 4

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \overset{\text{Player B}}{\begin{array}{cc} B_1 & B_2 \\ \end{array}} \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

(b) Prove that if a fixed number be added to each element of a pay-off matrix of a rectangular game, then the optimal strategies remain unchanged while the value of the game will be increased by that number. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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