WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

PHSADSE01T-PHYSICS (DSE1/2)

ADVANCED MATHEMATICAL PHYSICS I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
Answer mu be precise and to the point to earn credit
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Show that Laplace transformation is a linear transformation.

(b) Find the Laplace transform $\mathcal{L}\{f(t)\}$ of the function $f(t)=\begin{cases}1 \text{ for } t\geq t_0\\ 0 \text{ for } t\leq t_0\end{cases}$, t_0 being a constant.

(c) If $\tilde{f}(s)$ is the Laplace transform of $f(t)$, show that multiplying $f(t)$ by e^{at} moves the origin of s by and amount a.

(d) Find the Laplace transform of $f'(t)$ in terms of that of $f(t)$.

(e) From the definition of Laplace transform, show that $\mathcal{L}\{f(\omega t)\}=\frac{1}{\omega}\tilde{f}\left(\frac{s}{\omega}\right)$, for $\omega>0$, where $\tilde{f}(s)$ is the Laplace transform of $f(t)$.

(f) Show that the vectors $|a\rangle=(2,1,1)$; $|b\rangle=(-2,1,2)$; $|c\rangle=(0,0,1)$ are linearly independent.

(g) What is the significance of a non-singular transformation in a vector space?

(h) If a vector $|a\rangle\in\mathbb{R}^3$ in standard basis is $|a\rangle=(3,2,-1)$, find its components in the new basis (1, 1, 1), (1, 1, 0), (1, 0, 0).

(i) When are two finite dimensional vector spaces V and V' isomorphic?

(j) Show that members of an orthogonal set of vectors are linearly independent of each other.

(k) In the relation $L=I\omega$, show using quotient law that I is a tensor of rank two if L and ω are tensors of rank one.

(l) Prove the vector identity $\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}.\vec{C})\vec{B}-(\vec{A}.\vec{B})\vec{C}$ using properties of Levi-Civita symbol ε_{ijk}.

(m) Show that in three dimensions any anti-symmetric second rank tensor A_{ij} can be expressed in terms of a dual vector.

(n) If v_i are the components of a first rank tensor, show that $\frac{\partial v_i}{\partial x_i}$ is a tensor of rank zero.

2. (a) Find the Laplace transform of the periodic saw-tooth function with period T defined by 4+3+3

$$V(t) = V_0 \frac{t}{T}, \text{ for } 0 \le t \le T.$$

(b) In a two-dimensional complex linear vector space, $\{|a\rangle, |b\rangle\}$ from a non-orthogonal normalized basis where inner product of $|a\rangle$ and $|b\rangle$ is given by k. Form another orthonormal basis where $|c\rangle = \alpha|a\rangle + \beta|b\rangle$ is a basis element.

(c) From the defining properties of Krönecker delta, show that it is an invariant second-rank Cartesian tensor.

3. (a) Using tensorial symbols, establish the following vector identity: 4+4+2

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = \left(\vec{B}.\nabla\right)\vec{A} - (\vec{A}.\vec{\nabla})\vec{B} + A(\vec{\nabla}.\vec{B}) - B(\vec{\nabla}.\vec{A}).$$

(b) For a 3-dimensional vector space with an orthonormal basis formed by $|1\rangle$, $|2\rangle$ and $|3\rangle$, An operator $\hat{A}$ is defined by the relations: $\hat{A}|1\rangle = |2\rangle$, $\hat{A}|2\rangle = |1\rangle$ and $\hat{A}|3\rangle = |3\rangle$.

Find the matrix representation of A, using this basis. Is this operator unitary?

(c) Prove that the negative gradient of a scalar field is a first rank tensor.

4. (a) Show that $\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = s^2 \tilde{f}(s) - sf(0+) - \frac{df}{dt}(0+)$, where $\tilde{f}(s) = \mathcal{L}\{f(t)\}$. 3+3+(3+1)

(b) Using techniques of linear transformation, decouple the following coupled first order differential equations: $\frac{dx}{dt} = -ay$, $\frac{dy}{dt} = ax$.

(c) Consider a collection of rigidly connected N particles, rotating about an axis through the origin with angular velocity $\vec{\omega}$. If the particles, positioned at $\vec{r}_\alpha$, have mass m_α (where $\alpha = 0, 1, 2, ..., N$), find the component I_{ij} of the inertia tensor. Find the number of independent components of this tensor.

5. (a) From Newton's law for impulsive force acting on a particle of mass m, initially resting at the origin, the governing equation of its displacement $x(t)$ is written as $m\frac{d^2x}{dt^2} = P\delta(t)$, where P is a constant denoting the strength of the impulsive force. Show that the Laplace transform of $x(t)$ may be written as $\tilde{x}(s) = \frac{P}{ms^2}$. 4+4+2

(b) Apply Gram-Schmidt process to obtain an orthonormal set for the given vectors

$$\vec{A} = (-1, 0, 1);\ \vec{B} = (1, -1, 0);\ \vec{C} = (0, 0, 1) \text{ in } \mathbb{R}^3.$$

(c) Find the value of $\varepsilon_{ijk}\,\varepsilon_{ijk}$, where ε_{ijk} is the antisymmetric Cartesian tensor in three dimensions.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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