WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Consider a system whose Hamiltonian is given by $\hat{H}=\alpha(|\phi_1\rangle\langle\phi_2|+|\phi_2\rangle\langle\phi_1|)$, where α is a real number having appropriate dimension and $|\phi_1\rangle, |\phi_2\rangle$ are normalized eigenstates of an Hermitian operator $\hat{A}$ that has no degenerate eigenvalue. Determine if $|\phi_1\rangle, |\phi_2\rangle$ are eigenstates of $\hat{H}$.

(b) State Heisenberg uncertainty principle.

(c) Find the lowest energy of an electron confined to move in a one dimensional box of length 1 Å.

[Given, $m=9.11\times10^{-31}\text{kg}$, $\hbar=1.05\times10^{-34}\text{Js}$, $1\,\text{eV}=1.6\times10^{-19}\,\text{J}$]

(d) Consider the operator $\hat{Q}=i\frac{d}{d\phi}$, where ϕ is plane polar azimuthal angle in two dimensions. Write down its eigenvalue equation and find its eigenvalues.

(e) What is Larmor precession of an electron in an atom?

(f) For one dimensional bound state motion of a particle of mass *m*, prove that the expectation value of its kinetic energy is given by $<K>=\frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\frac{\partial\psi}{\partial x}\frac{\partial\psi^*}{\partial x}dx$.

(g) If $\psi_1(x,t)$ and $\psi_2(x,t)$ are both solutions of the time dependent Schrödinger equation for the motion of a particle with potential energy $V(x)$, prove that the linear combination $\psi(x,t)=A_1\psi_1(x,t)+A_2\psi_2(x,t)$ is also a solution, where A_1 and A_2 are constants.

(h) Calculate the probability current density of a quantum mechanical system of mass *m* and described by the state function $\psi(r)=\frac{1}{r}e^{ikr}$.

(i) A particle constrained to move along x-axis in the domain $0 \le x \le L$ has a wavefunction $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(n\frac{\pi x}{L}\right)$, where n is an integer. What is the expectation value of its momentum?

(j) Plot the wavefunctions of the ground state and the 1st excited state for a particle within an infinite square well potential of width a. Also plot the probability density for those states.

(k) Find the probability of finding an electron within Bohr radius for the ground state of hydrogen atom. Given that the ground state wavefunction is $\psi_{1S} = \sqrt{\frac{1}{\pi a^3}}\exp\left(-\frac{r}{a}\right)$, where a is the Bohr radius.

(l) Find the degeneracy of the n-th energy eigenstate for the electron in a hydrogen atom neglecting its spin.

(m) In a Stern-Gerlach experiment, on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam?

(n) Find the uncertainty in the measurement of S_z on a system prepared in a state $|\psi\rangle = \frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenvectors of the operator $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively.

2. A particle of mass m is in a normalized state $\psi(x) = Ae^{-a\left[\frac{mx^2}{\hbar}+it\right]}$, where A and a are constants.

(a) Find A. 2

(b) For what potential energy function $V(x)$ does $\psi(x)$ satisfy Schrödinger equation. 2

(c) Calculate the expectation values of x and p_x. 3+3

3. (a) Starting from the time-dependent Schrödinger equation in one dimension, derive the equation of continuity of probability. 4

(b) If a dynamical variable is represented by the quantum mechanical operator $\hat{A}$ that does not depend on time explicitly, prove $\frac{d}{dt}\langle\hat{A}\rangle = \frac{i}{\hbar}[\hat{H}\hat{A} - \hat{A}\hat{H}]$, where $\hat{H}$ is the Hamiltonian operator. 3

(c) If $\{|\vec{r}\rangle\}$ and $\{|\vec{p}\rangle\}$ represent the bases in position and momentum representations respectively, then prove that $\langle\vec{p}\,|\,\vec{r}\rangle = \langle\vec{r}\,|\,\vec{p}\rangle^* = \frac{1}{(2\pi\hbar)^{3/2}}e^{-i(\vec{p}.\vec{r})/\hbar}$ 3

4. (a) Ground state eigenfunction of a one dimensional quantum oscillator is 3

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{\left\{-\frac{m\omega}{2\hbar}x^2\right\}}.$$

Show that its ground state energy is $E_0 = \frac{1}{2}\hbar\omega$

(b) Write down the Schrödinger equation in spherical polar co-ordinates corresponding to the hydrogen atom problem. Write down the radial part of the wave function. 2

(c) What is space quantization of orbital angular momentum? 2

(d) Find the normalized ground state wavefunction of linear harmonic oscillator using the operator $\hat{a} = \frac{m\omega\hat{x} + ip}{\sqrt{2m\omega\hbar}}$. 3

5\. (a) Explain the term 'spin-orbit coupling'. 2

(b) In a many electron atom, the orbital, spin and total angular momenta are denoted by $L = 2$, $S = 1$ and $J = 2$. Find the angle between $\vec{L}$ and $\vec{S}$ using the vector atom model. 2

(c) Derive an expression for the magnetic moment for an electron moving in a circular orbit. Hence show that the ratio of the orbital magnetic moment to its angular momentum is $\frac{e}{2m}$. 3+3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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