



**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 3rd Semester Examination, 2020, held in 2021

**STSACOR06T-STATISTICS (CC6)**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**GROUP-A**

**Answer any four questions from the following**

**$5 \times 4 = 20$**

1. Let  $\{X_1, X_2, \dots, X_n\}$  be a random sample from a distribution with  $E(X_1) = \mu$  and  $Var(X_1) = \sigma^2$ . Show that 2+3

$$T = \frac{1}{2n(n-1)} \sum_{i \neq j} (X_i - X_j)^2$$

is an unbiased estimator of  $\sigma^2$ . Also show that  $T$  is equivalent with sample variance of  $\{X_1, X_2, \dots, X_n\}$ .

2. Let  $X$  and  $Y$  are iid  $\mathcal{N}(0, 1)$ . Find the distribution of 3+2

$$(i) U = \frac{X}{|Y|} \quad \text{and} \quad (ii) V = \frac{X}{|X|}.$$

3. Define Chi-square distribution with  $n$  degrees of freedom ( $\chi_n^2$ ). Find its moment generating function. Show that for any  $c > 0$ ,  $P(\chi_n^2 > c)$  increases with  $n$ . 1+2+2

4. The life (in hours) of an electronic device of a certain type has the exponential distribution with mean  $1/\lambda$ . Let five devices of this type be tested simultaneously. 3+2

- (i) At what time should we expect the first one of them to fail?  
(ii) At what time should we expect all of them to fail?

5. Define simple and composite hypothesis. Discuss how you test for the homogeneity of two independent Poisson populations. 1+4

6. Distinguish between test procedures based on P-value approach and fixed level approach. 5

**GROUP-B**

**Answer any two questions from the following**

**$10 \times 2 = 20$**

7. (a) Discuss the importance of orthogonal transformation in Statistics. 3+7
- (b) Let  $\{X_1, X_2, \dots, X_{2n}\}$  be a random sample from  $\mathcal{N}(\mu, \sigma^2)$ . Derive the sampling distribution of  $(X_1 + X_2 + \dots + X_n - X_{n+1} - X_{n+2} - \dots - X_{2n})/2n$ .
8. Derive sampling distribution of Student's t-statistic. Discuss its use. 8+2
9. (a) What are the errors associated with testing of statistical hypothesis? 3+7
- (b) Suppose  $\{X_1, X_2, \dots, X_n\}$  be a random sample from  $\mathcal{N}(\mu, 4)$  population. For the testing problem  $H_0: \mu = 2$  vs.  $H_1: \mu = 4$ , consider the following critical regions:
- (i)  $X_1 > 4.7$
  - (ii)  $\frac{1}{3}(X_1 + 2X_2) > 4.5$
  - (iii)  $\frac{1}{2}(X_1 + X_3) > 4.2$
  - (iv)  $\bar{X} > 4.1$ .

Find type I and type II errors probabilities for each of the tests and compare.

**N.B. :** Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

