लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2020, held in 2021

STSACOR05T-STATISTICS (CC5)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 5×4 = 20

1. If $P(X=x,\ Y=y)=\frac{1}{3}$, $(x,y)=(0,2),\ (2,0),\ (1,1)$ and $P(X=x,\ Y=y)=0$ for any other $(x,\ y)$, find the regression equations and the correlation coefficient. Comment on your findings. 5

2. If $(X,\ Y)$ follows Bivariate Normal, $BN(0,\ 0,\ \sigma^2,\ \sigma^2,\ \rho)$, find the joint distribution of $U=aX+bY$ and $V=cX+dY$ for all $a,\ b,\ c,\ d,\ ad-bc\neq 0$. 5

   When are the random variables U and V independently distributed?

3. Obtain the moment generating function of Cauchy distribution. 5

4. Let $X\sim U(0,\ 1)$ and $Y\mid X\sim \text{Bin}(n,\ p)$. Find the distribution of Y. Also find $E(Y)$. 5

5. Find the mean, variance and skewness of the following distribution. 5

$$f(x)=\frac{1}{2}e^{-|x|},\ x\in\mathbb{R}$$

6. Define Probability generating function (PGF) and Characteristic function (CF). Compute PGF of binomial random variate. $1\frac{1}{2}+1\frac{1}{2}+2$

GROUP-B

Answer any *two* questions from the following 10×2 = 20

7. (a) A continuous random variable Y has the following probability density function: 3+2+5

$$f_Y(y) = \begin{cases} 0 & , \text{ if } \quad y < 0 \\ 3(1-\frac{y}{k}) & , \text{ if } \quad 0 \le y \le k \\ 0 & , \text{ if } \quad y > k \end{cases}$$

Find k and therefore $\text{Var}(Y)$.

(b) Prove memory loss property of Exponential distribution.

(c) If $X \sim$ Lognormal Distribution $\Lambda(\mu, \sigma^2)$ show that

$\text{mean}(X) > \text{median}(X) > \text{mode}(X)$.

8. (a) Find MGF of a normal distribution with mean θ and variance σ^2. Show that (4+3)+3

$$\mu_{2r} = (2r-1)\mu_{2r-2}\sigma^2$$

(b) Show that normal distribution may be looked upon as a limiting distribution of binomial distribution.

9. (a) If $e^{-t^2/2}$ is the characteristic function of a random variable x, then show that the pdf of x is $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. 5+5

(b) If $x_1, x_2, \cdots, x_n$ are independent random variables all having same pdf $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, show by using characteristic functions that the random variable $\frac{1}{\sqrt{n}}(x_1 + x_2 + \cdots + x_n)$ also has the same pdf.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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