WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

STSACOR02T-STATISTICS (CC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions 5×4 = 20

1. For any two non-zero real numbers u and v, show that there exists a number $n \in \mathbb{N}$ such that $n|u| > v$, where $\mathbb{N}$ is the set of natural numbers. 5

2. What is Cauchy sequence? Show that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. 1+4

3. Define Ratio test and Root test in connection with series of real numbers. Check for the convergence of the series $\sum_{n \geq 1} \left(1 - \frac{1}{n}\right)^{n^2}$. 3+2

4. Consider the following partition matrix: 5

$$\left[\begin{array}{c:cccc} np_1q_1 & -np_1p_2 & -np_1p_3 & \cdots & -np_1p_k \\ \hline -np_2p_1 & np_2q_2 & -np_2p_3 & \cdots & -np_2p_k \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ -np_kp_1 & -np_kp_2 & -np_kp_3 & & np_kq_k \end{array}\right],$$

where $p_i + q_i = 1$, $p_i > 0$, for all $i = 1, 2, \cdots, k$, and $\sum_{i=1}^{k} p_i = 1$. Using the partition find the determinant of the above matrix.

5. Prove that for any two square matrix $A^{n \times n}$ and $B^{n \times n}$, $|AB| = |A||B|$. 5

6. Let $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_n\}$ be a set of n linearly independent n-component vectors. Show that, for every n-component vector $\boldsymbol{\beta}$, the set $\{\boldsymbol{\beta}, \boldsymbol{\alpha}_1, \cdots, \boldsymbol{\alpha}_n\}$ is linearly dependent. 5

GROUP-B

Answer any *three* questions 10×3 = 30

7. (a) State Cauchy's first theorem on limit. Hence show that if the series $\sum_{n\geq 1} a_n$ is convergent then $\frac{1}{n}\sum_{k=1}^{n} k\, a_k \to 0$ as $n \to \infty$. 1+3

(b) Distinguish between absolute and conditional convergence. Show that, if a series $\sum_{n\geq 1} a_n$ of real numbers is convergent conditionally then $\sum_{n\geq 1} a_n^{+} = \infty$ and $\sum_{n\geq 1} a_n^{-} = \infty$ where $a_n^{+} = \max\,(a_n, 0)$ and $a_n^{-} = -\min\,(a_n, 0)$. 2+4

8. (a) State and prove Leibnitz Theorem in connection with real series. 6

(b) Provide examples of real sequences $\{a_n\}$ and $\{b_n\}$ such that 2+2

(i) both $\{a_n\}$ and $\{b_n\}$ are divergent but $\{a_n + b_n\}$ is convergent and $\{a_n - b_n\}$ is divergent;

(ii) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent but $\{a_n b_n\}$ is convergent and $\{\frac{a_n}{b_n}\}$ is divergent.

9. Show that all the vectors $(x_1, x_2, x_3)'$ in a vector space V_3 with restriction $x_1 - x_3 = x_2$ form a vector subspace V of V_3. Then show that V is spanned by $\boldsymbol{a_1} = (1, 0, 1)'$ and $\boldsymbol{a_2} = (0, 1, -1)'$. Find an orthonormal basis of V. Also find a vector which is orthogonal to all vectors in V. 3+2+3+2

10. (a) Let $\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \cdots, \boldsymbol{\alpha_r}$ be a basis of a vector space V. If $\boldsymbol{\beta} = C_1\boldsymbol{\alpha}_1 + \cdots + C_r\boldsymbol{\alpha}_r$ when $C_r \neq 0$, then show that $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_{r-1}, \boldsymbol{\beta}$ is also a basis for V. 6

(b) S is a skew-symmetric matrix and $(I - S)$ is non-singular. Show that $(I - S)(I + S)^{-1}$ is an orthogonal matrix. 4

11. (a) What is multiple root of a polynomial? Discuss its effect on the graph of the polynomial. 1+2

(b) Using appropriate methods, find all the roots of the polynomial function $P_4(x) = 8x^4 + 6x^3 + 11x^2 - 7x - 6$. 7

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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