WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2020

PHSACOR08T-PHYSICS (CC8)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) If $z=x+iy$, show that $|\sin z|^2=\sin^2 x+\sinh^2 y$.

(b) Evaluate $\oint f(z)dz$ for $f(z)=1/z$ along the circle of radius R centred at origin.

(c) Show that $f(z)=\sin z/z$ has a removable singularity at $z=0$.

(d) Find the branch points of $f(z)=\sqrt{(z^2+1)}$.

(e) $f(z)=u(x,y)+iv(x,y)$ is analytic where $u=x^2-y^2$. Find v.

(f) Prove that if $f(x)$ is periodic with period a then Fourier transform $\tilde{f}(k)=0$ unless $ka=2\pi n$ for n being an integer.

(g) If Fourier transform of $f(x)$ is $g(s)$, then show that Fourier transform of $f(x)\cos ax$ is $\frac{1}{2}[g(a+s)+g(a-s)]$.

(h) Find Fourier transform of a Dirac delta function $f(x)=\delta(x-b)$, b being some constant.

(i) What kind of boundary condition do you need for unique solution of Laplace equation in a bound smooth domain?

(j) Show that real and imaginary parts of an analytic complex function individually satisfy Laplace's equation in two dimensions.

(k) For a 2 × 2 square matrix A find its eigenvalues in terms of t and d, given $\mathrm{Tr}(A)=t$ and $\det(A)=d$.

(l) Prove that the product of two Hermitian matrices is Hermitian if and only if they commute.

(m) Find eigenvalues of matrix $\boldsymbol{R}=\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$.

(n) Pauli spin matrix σ_x is conventionally written as, $\sigma_x=\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$. Find $\sin\alpha\sigma_x$, α being a constant.

2. (a) State with justification, whether or not the function $f(z)=Re(z)=x$ is analytic. Find the Laurent Series of $f(z)=\dfrac{1}{z(z-2)^3}$ about the singularity $z=2$ and find the residue of $f(z)$ at $z=2$. 1+2+1

(b) Solve using Fourier Transform $\dfrac{d^2\phi}{dx^2} - m^2\phi = f(x)$, in terms of an integral, m being some constant. 2

(c) Verify Caley-Hamilton theorem for the matrix $\boldsymbol{A} = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, and hence find $\boldsymbol{A}^{-1}$. 2+2

3. (a) Expand $f(z) = \dfrac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$. 2

(b) In physical optics, Fraunhofer diffraction pattern is given by Fourier transform of the aperture function. Suppose the aperture function (for a single slit), 2+2

$$f(x) = \begin{cases} 1, & |x| < a, \\ 0, & |x| > a, \end{cases}$$

Calculate $F(t)$, the amplitude of the diffraction pattern. Use Parseval relation to calculate

$$\int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2}\, dt\,.$$

(c) An uncharged conducting sphere of radius R is placed in a uniform electrostatic field $\vec{E} = E_0\hat{k}$. Find the potential outside the sphere using solution of Laplace's equation in spherical polar coordinates. 4

4. (a) Evaluate the following integral, 3

$$I = \oint_C \frac{z+1}{z^4 + 2iz^3}\, dz\,,$$

where C is the circle $|z| = 1$.

(b) What is meant by the Fourier transform of a function $f(x)$? Show that under complex conjugation Fourier transform of a real function $f(x)$ satisfies $\tilde{f}(-k) = [\tilde{f}(k)]^*$. 1+2

(c) Solve one dimensional heat equation, 4

$$\frac{\partial u(x,\, t)}{\partial t} = \frac{\partial^2 u(x,\, t)}{\partial x^2}\,,$$

for $t > 0$ and $u(x, 0) = \delta(x)$.

5. (a) Evaluate the integral 3

$$I = \int_0^{2\pi} \frac{\mathrm{d}\theta}{5 + 4\cos\theta}$$

(b) Find the eigenvalues of $\boldsymbol{H} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. 3+4

Also show that its diagonalizing matrix (which makes it diagonal by similarity transformation) can be chosen to be orthogonal.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——