WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2020

STSHGEC02T/STSGCOR02T-STATISTICS (GE2/DSC2)

INTRODUCTION TO PROBABILITY

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer any *four* questions from the following 5×4 = 20

1. For two random variables X and Y, $E(X)=8$, $E(Y)=6$, $\text{var}(Y)=36$ and $r_{XY}=0.5$.

   Find (i) $E(XY)$,

   (ii) $\text{cov}(X, X+Y)$,

   (iii) $\text{var}(2X-2Y)$

2. Define probability density function of a random variable X. Is the following a probability density function?

$$f(x) = \begin{cases} x/2, & 0 < x \le 1 \\ \frac{1}{2}, & 1 < x \le 2 \\ \frac{3-x}{2}, & 2 < x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

3. Suppose $P(A)=p_1$, $P(B)=p_2$ and $P(A\cap B)=p_3$. Show that $P(A^c\cap B^c)=1-p_1-p_2+p_3$.

4. State Weak Law of Large Numbers (WLLN). Determine whether it holds for the following sequence of independent random variables:

$$P(x_n=+1)=\frac{1}{2}(1-2^{-n})=P(x_n=-1)$$

5. For a Binomial distribution with parameters n and p, establish the following relationship $\mu_{r+1}=pq\left(nr\mu_{r-1}+\frac{d\mu_r}{dp}\right)$

6. For a normal distribution with mean 3 and variance 16, find the value of y of the variate such that the probability of the variate lying in the interval (3, y) is 0.4772. [You are given $P(Z \le 2) = 0.9772$].

Answer any *two* questions from the following 10×2 = 20

7. (a) For mutually exclusive events $A_1, A_2, \ldots, A_n$, prove that $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$. What will be the value of $P(\bigcup_{i=1}^{n} A_i)$ if $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events. 5+5

(b) Let $B_1, B_2, \ldots, B_n$ be exhaustive and mutually exclusive events with $P(B_i) > 0$, $i = 1, 2, \ldots, n$. Show that for any event A, $P(A) = \sum_{i=1}^{n} P(B_i) P(A \mid B_i)$.

8. A random variable X has pmf $f(x) = \begin{cases} \frac{x}{21}, & \text{for } x = 1, 2, \ldots, 6 \\ 0, & \text{otherwise} \end{cases}$. 10

Find $P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right)$.

9. (a) Write down the p.d.f of Normal (μ, σ^2) distribution. Show that this distribution is symmetric. Calculate its median and mode. 5+5

(b) Find the mean deviation about mean of X where $X \sim N(0, \sigma^2)$.

10.(a) Define CDF of a random variable. Write down the properties to be satisfied by a CDF. 5+5

(b) For a Binomial distribution prove that $\text{cov}(X, n - X) = -npq$, where the notations have their usual meaning.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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