WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2020

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

DIFFERENTIAL EQUATIONS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Examine whether $\{\cos x \tan y + \cos(x+y)\}dx + \{\sin x \sec^2 y + \cos(x+y)\}dy$ is an exact differential equation.

(b) Show that the functions 1, x and x^2 are linearly independent. Hence find the differential equation whose solutions are 1, x and x^2.

(c) Prove that if f and g are two different solutions of $y' + P(x)y = Q(x)$, then $f - g$ is a solution of the equation $y' + P(x)y = 0$.

(d) Show that $\{x(x^2 - y^2)\}^{-1}$ is an integrating factor of the differential equation $(x^2 + y^2)dx - 2xydy = 0$.

(e) Find a particular integral of the differential equation

$$(D^2 - 4D)y = x^2 \text{ where } D \equiv \frac{d}{dx}.$$

(f) Eliminating the arbitrary constants from the following equation form the partial differential equation:

$$z = (a + x)(b + y)$$

(g) Eliminate the arbitrary function f and g from $z = f(x+iy) + g(x-iy)$ where $i^2 + 1 = 0$.

(h) Find the order and degree of the following differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + x^2\left(\frac{dy}{dx}\right)^4 = 4$$

2. (a) Obtain the general solution of the differential equation — 4

$$xdy - ydx + a(x^2 + y^2)dx = 0$$

(b) Determine the constant A so that the following differential equation is exact and hence solve the resulting equation: 4

$$\left(\frac{Ay}{x^3}+\frac{y}{x^2}\right)dx+\left(\frac{1}{x^2}-\frac{1}{x}\right)dy=0$$

3\. (a) Given that $y=x+1$ is a solution of $[(x+1)^2D-3(x+1)D+3]y=0$, find a linearly independent solution by reducing the order. Hence determine the general solution. $\left(D\equiv\frac{d}{dx}\right)$ 5

(b) Find an integrating factor of the following differential equation 3

$$x\frac{dy}{dx}+\sin 2y=x^4\cos^2 y$$

4\. (a) Obtain complete primitive and singular solution of 3

$$y=px+(1+p^2)^{1/2}$$

(b) Solve: $p^2+px=xy+y^2$ 5

5\. (a) Show that e^x and xe^x are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0)=1$, $y'(0)=4$. 1+1+1+1+1

Is it unique solution? Over which interval is it defined?

(b) The complementary function of $\frac{d^2y}{dx^2}+y=\cos x$ is $A\sin x+B\cos x$, where A and B are constants. Find a particular integral. 3

6\. (a) Apply the method of variation of parameters to solve the following equation: 6

$$x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^2\log x$$

(b) Fill in the blank: 2

In the 'method of variation of parameter' if $y=A\ f_1(x)+B\ f_2(x)$ be the complementary function then the complete primitive is $y=\phi(x)f_1(x)+\psi(x)f_2(x)$ provided …………..

7\. (a) Solve: $\frac{dx}{dt}=-2x+7y$, $\frac{dy}{dt}=3x+2y$ subject to the conditions $x(0)=9$ and $y(0)=-1$. 4

(b) Solve: $\frac{d^2y}{dx^2}+y=\sin 2x$ given that $y=0$ and $\frac{dy}{dx}=0$ when $x=0$. 4

8. (a) Verify that the following equation is integrable and find its primitive: 5

$$zydx+(x^2y-zx)dy+(x^2z-xy)dz=0.$$

(b) Find a complete integral of the following partial differential equation by Charpit's method: $z=p+q$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. 3

9. (a) Find the particular solution of the differential equation 5

$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y$ which passes through the curve $xy=4$, $z=0$.

(b) Classify the partial differential equation 3

$$\frac{\partial^2 z}{\partial x^2}+(1-x)\frac{\partial^2 z}{\partial y^2}=0$$

into elliptic, parabolic and hyperbolic for different values of x.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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