



## WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2020

### STSACOR04T-STATISTICS (CC4)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

#### GROUP-A

**Answer any four from the following questions**

5×4 = 20

1. Define limit of a real valued function. Let  $f : D \rightarrow \mathbb{R}$  and  $g : D \rightarrow \mathbb{R}$  be two functions and  $c$  be a cluster point of  $D$  such that  $f(x) \leq g(x)$  for all  $x \in D - \{c\}$ . Show that  $\lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} g(x)$ . 1+4
2. What do you mean by a locally bounded function? Show that  $f(x) = \frac{1}{x}$  is not bounded on  $(0, 1)$  but it is locally bounded at every point on  $(0, 1)$ . 1+4
3. Define continuity of a function at a point. Distinguish between jump discontinuity and removable discontinuity with appropriate examples. 1+4
4. State Rolle's Theorem. Construct a function (with appropriate justification) such that the function is continuous at a single point but nowhere differentiable. 2+3
5. Show that the sequence  $\{e^{-nx}\}$  of functions converges pointwise on  $[0, \infty)$  but uniformly on  $[a, \infty)$  for any  $a > 0$ . 5
6. Define characteristic roots and vectors of a matrix. "Each eigenvalue of a real skew-symmetric matrix  $A$  is either 0 or a purely imaginary number" — Justify the statement. 2+3

#### GROUP-B

**Answer any three from the following questions**

10×3 = 30

7. (a) Define power series. Describe the concept of radius of convergence with an example. 2+2

- (b) State Taylor's Theorem with Lagrange's form of remainder. Find Taylor's series of the function  $\ln x$  about  $x_0 = 1$ . 2+4
8. (a) Define uniform continuity of a function. Show that  $f(x) = \sqrt{x + \frac{3}{8}}$  is uniformly continuous on  $[-\frac{3}{8}, \infty)$ . 1+5
- (b) Show that  $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$  is convergent if  $m > 0$  and  $n > 0$ . 4
9. (a) State Fubini's Theorem in connection with double integration. Find  $\iint_{\Delta} e^{-(x^2+y^2)} dx dy$  2+4
- where  $\Delta$  is the region bounded between the curves  $x^2 + y^2 = 2$  and  $x^2 + y^2 = 5$ .
- (b) Define derivative of a function. Hence show that if  $f: \mathbb{R} \rightarrow \mathbb{R}$  is an even function and differentiable on  $\mathbb{R}$  then the derivative  $f'(x)$  is an odd function on  $\mathbb{R}$ . 2+2
- 10.(a) Define the rank of a matrix. Let  $A$  and  $B$  be two matrices so that  $AB$  is defined. Then show that  $\text{Rank}(AB) \leq \text{Rank}(A)$  and if  $B$  is non-singular then  $\text{Rank}(AB) = \text{Rank}(A)$ . 2+5
- (b) Reduce the following  $3 \times 3$  matrix in Echelon form and hence comment on its rank. 3
- $$\begin{bmatrix} 2 & 5 & 18 \\ 6 & 35 & 5 \\ 11 & 2 & 7 \end{bmatrix}$$
- 11.(a) Define a system of linear equations. Discuss the condition for consistency of a system of linear equations. Also provide the conditions for the existence of unique solution and infinite solutions. 1+4+2
- (b) Show that, for a matrix  $B$  of order  $n \times n$ , the symmetric matrix  $B^T B$  is either positive definite or positive semi-definite. 3

**N.B. :** Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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