WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2020

MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Show that $f(x,y)=x^2\cos^2 y+y\sin^2 x$ satisfies Lipschitz condition on $|x|\le 1,\ |y|<\infty$, and find a Lipschitz constant.

(b) Solve: $(D^3+3D^2+3D+1)y=0$, $D\equiv\dfrac{d}{dx}$.

(c) Find a particular integral for

$$(D^2+1)y=\sin 2x,\ \ D\equiv\frac{d}{dx}.$$

(d) Determine whether $x=-1$ is an ordinary point or a regular singular point of the differential equation: $x^2(x+1)^2\dfrac{d^2y}{dx^2}+(x^2-1)\dfrac{dy}{dx}+2y=0$.

(e) Find a fundamental matrix for the system $\dot{\mathbf{X}}(t)=\mathbf{A}\mathbf{X}(t)$, where $\mathbf{A}=\begin{pmatrix}4 & 3\\ 2 & 5\end{pmatrix}$, $\mathbf{X}(t)=\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix}$, and $\bullet$ denotes differentiation with respect to t.

(f) Find the constant λ such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar.

(g) A particle moves along the curve $x=2t^2$, $y=t^2-4t$, $z=-t-5$, where t is the time. Find the component of velocity at time $t=1$ in the direction $\hat{i}-2\hat{j}+2\hat{k}$.

(h) If $\phi(x,y,z)=3x^2yz$ and C is the curve $x=t^2$, $y=t^3$, $z=t$, from $t=0$ to $t=1$, evaluate the vector line integral $\int_C \phi\, d\vec{r}$.

2. (a) Solve: $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+y=\dfrac{\log x\sin\log x+1}{x}$. 4+4

(b) Show that $e^x\sin x$ and $e^x\cos x$ are linearly independent solutions of $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+2y=0$. State the general solution and find the solution satisfying the conditions $y(0)=2,\ y'(0)=3$.

3. (a) Show that the differential equation $\frac{dy}{dx}=1+y^2$, $y(0)=0$ has unique solution in some interval about $x=0$. 4+4

(b) Show that the Wronskian of two solutions y_1 and y_2 of $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ satisfies $\frac{dW}{dx}+P(x)W=0$.

Also show that if y_1 is known then y_2 can be obtained in the form $y_2=y_1\int\frac{W(y_1,y_2)}{y_1^2}dx$

4. (a) Solve by the method of undetermined coefficients: 4+4

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=24e^{-3x}$$

(b) Find the solution $\mathbf{X}(t)=(x_1(t),\ x_2(t))^T$ such that $\mathbf{X}(0)=(1,6)^T$, for the system

$$\frac{dx_1}{dt}=2x_1-x_2$$
$$\frac{dx_2}{dt}=\quad -4x_2$$

5. Solve by the method of variation of parameters: 4+4

(i) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$,

with $y(0)=0$, and $\left(\frac{dy}{dx}\right)_{x=0}=0$.

(ii) $\frac{d^2y}{dx^2}+a^2y=\sec ax$.

6. (a) Solve: 4+4

$$(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2.$$

(b) Solve: $(D^4+D^2+1)y=e^{-\frac{x}{2}}\cos\frac{\sqrt{3}x}{2}$.

7. (a) Locate and classify the singular points of the differential equation: 3+5

$$x^3(x^2-1)\frac{d^2y}{dx^2}+2x^4\frac{dy}{dx}+4y=0$$

(b) Find the series solution near $x=0$ of the equation

$$x^2\frac{d^2y}{dx^2}+(x+x^2)\frac{dy}{dx}+(x-9)y=0$$

8. (a) Find the equations of the tangent, principal normal and binormal of the space curve: 5+3

$$\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\,\hat{k} \text{ at } t = \pi .$$

(b) Show that the following points are coplanar by using the box-product:

$$A(-1, 1, 2),\ B(1, -2, 1),\ C(2, 2, 4),\ D(-2, 0, 1).$$

9. (a) If $\dfrac{d^2\vec{a}}{dt^2} = 6t\,\vec{i} - 24t^2\,\vec{j} + 4\sin t\,\vec{k}$, find $\vec{a}$, given that, $\vec{a} = 2\vec{i} + \vec{j}$ and $\dfrac{d\vec{a}}{dt} = -\vec{i} - 3\vec{k}$ at $t = 0$. 4+4

(b) Show that $\vec{a} \times \dfrac{d\vec{b}}{dt} = \vec{b} \times \dfrac{d\vec{a}}{dt}$, and give a geometrical interpretation of the result.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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