WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2020

MTMACOR03T-MATHEMATICS (CC3)

REAL ANALYSIS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $S=\left\{\frac{1}{n}: n\in N\right\}$, show that $\inf S = 0$. 2

(b) Find the interior of N, the set of all natural numbers. Is N an open set? Give reasons in support of your answer. 1+1

(c) Define isolated point of $S\subset R$, where R denotes the set of all real numbers. Find the isolated points of the set Q of rational numbers. 1+1

(d) Justify: Sum of two non-convergent sequences $\{x(n)\}$ and $\{y(n)\}$ may or may not be convergent. 1+1

(e) Two sets A and B or real numbers are such that A is closed and B is compact. Prove that $A\cap B$ is compact. 1+1

(f) Show that the sequence $\{f(n)\}$ defined by 1+1

$f(n)=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+................+\frac{1}{n.(n+1)}$ is monotonically increasing and bounded above.

(g) Test the convergence of the series: $\sum_{n=1}^{\infty}(-1)^{n-1}n^{\frac{1}{2}}$. Examine its absolute convergence. 2

(h) Test the convergence of the series: $\sum_{n=1}^{\infty}(1+\frac{1}{n})^{-n^2}$ 2

2. (a) State Archimedean property of R, the set of real numbers and use this to prove that if $x\in R$ and $x>0$, there exists a natural number n such that $0<\frac{1}{n}<x$. 1+1

(b) Prove that the set Q of rational numbers is Archimedean. 2

(c) State the supremum property of R. Show that the supremum property is not satisfied by Q, the set of rational numbers. 1+3

3. (a) If $G \subset R$ be an open set in R and $F \subset R$ be a closed set in R. Then prove that $G - F$ is an open set in R. 2

   (b) Show by example that the union of an infinite number of closed sets in R is not a closed set in R. 2

   (c) Give example of an infinite subset of R which has no limit point. Give reasons. 2

   (d) If $A = (2, 3)$ and $B = (3, 4)$, then verify whether $d(A \cap B) = d(A) \cap d(B)$ or not, where $d(A)$ denotes the derived set of A. 2

4. (a) Let $S \subset R$, the set of real numbers. Define an open cover of S and a subcover of S. 1+1

   (b) Use the definition of a compact set to prove that the intersection of two compact sets in R is compact. 2

   (c) If $A = (-1, 1)$, $B = \{2, 3, 1, -1\}$ then correct or justify the statement: $A \cup B$ is compact. 2

   (d) Give an example of a set in R which is closed but not compact in R. Give reasons. 2

5. (a) Define an enumerable set and a countable set. 1+1

   (b) Prove that the set of integers is enumerable. 2

   (c) Prove that a subset of an enumerable set is either finite or enumerable. 4

6. (a) Prove that a monotonically decreasing sequence which is bounded below is convergent and it converges to the greatest lower bound. 2

   (b) Find the bounds of the sequence $\{f(n)\}$ where $f(1) = 1$, 1+1

$$f(n) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots\dots + \frac{1}{(n-1)!}, n \geq 2.$$

   (c) If $u(n) = \dfrac{n!}{n^n}$, then prove that the sequence $\{u(n)\}$ is a null sequence. 2

   (d) Use Cauchy's 1st theorem on limit to find 2

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n^2+1}} + \frac{1}{\sqrt{2n^2+2}} + \dots\dots\dots\dots + \frac{1}{\sqrt{2n^2+n}}\right]$$

7. (a) State Bolzano-Weierstrass theorem for sequences. 1

   (b) Give examples to show that a bounded sequence may have a divergent subsequence and an unbounded sequence may have a convergent subsequence. 1+1

   (c) Find the upper limit and the lower limit of $\{f(n)\}$ where 2

$$f(n) = (1 - \frac{1}{n^2})\sin\frac{n\pi}{2}.$$

   (d) Justify: Every Cauchy sequence is bounded. Is the converse true? Give example to support your answer. 1+2

8. (a) Use comparison test (limit form) to examine the convergence of the series: 3

$$\frac{1}{4.6}+\frac{\sqrt{3}}{6.8}+\frac{\sqrt{5}}{8.10}+\ldots\ldots\ldots\ldots$$

(b) Show by Cauchy's Integral test that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges for $0<p<1$. 2

(c) Test the convergence of the series whose n-th term is given by 3

$$\frac{1.3.5.\cdots\cdots\cdots(2n-1)}{2.4.6.\cdots\cdots\cdots\cdots(2n)}.\frac{1}{n}$$

9. (a) Apply Ratio test to examine the convergence and divergence of the infinite series $\sum_{n=1}^{\infty}\frac{x^n}{n}$ in different ranges of values of $x\in R$ with $x>0$. 3

(b) Correct or justify: Every convergent series is absolutely convergent. Give reasons. 2

(c) Show that the alternating series $\sum_{n=1}^{\infty}(-1)^n\frac{n+2}{2^n+5}\cos nx$ is convergent for all real values of x. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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