WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-II Examination, 2020

MATHEMATICS

PAPER: MTMA-III

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *three* from other questions

1. Answer any ***four*** questions from the following: $2\times4 = 8$

(a) Find the least value of $x^2+y^2+z^2$ for positive values of x, y, z which satisfy $x+y+z=10$.

(b) Show that the derived set of $T=\left\{\left(\frac{1}{m}, \frac{1}{n}\right)\big/\ m, n \in \mathbb{N}\right\}$ is an infinite set.

(c) It is impossible for a system of linear equations to have exactly two solutions. Explain why.

(d) Give an example of an unbounded sequence with two subsequences one of which is convergent and other divergent.

(e) Show that $f(x)=\frac{1}{x}$ is not uniformly continuous over (0, 1) but is uniformly continuous over $[a, 1]$ where $0<a<1$.

(f) Define a reciprocal equation. Prove that the equation $(1+x)^5=a(1+x^5)$ is a reciprocal one where $a\neq 1$.

(g) Find an example of a non-cyclic group, all of whose proper subgroups are cyclic.

(h) Suppose A has eigen values 1, 2, 4. What is the trace of A^2? What is the determinant of $(A^{-1})^T$?

2. (a) Let $f(x, y)=g\left(\sqrt{x^2+y^2}\right)$, where $g(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0\end{cases}$ 5

Show that f is differentiable at (0, 0) but f_x, f_y are not continuous at (0, 0).

(b) If $f(x)\equiv x^4+x^3-4x^2-3x+3$, solve $f(x-2)=0$. 5

(c) Extend the set of vectors {(–3, 2, –1), (1, –1, –5)} to an orthogonal basis of the Euclidean space $\mathbb{R}^3$ with standard inner product. Find the associated orthonormal basis. 4

3. (a) Prove that every continuous function defined over a closed and bounded domain in $\mathbb{R}^2$ is bounded on its domain. 4

(b) Find the product of inertia of a semicircular wire of radius a and mass M about diameter and tangent at its extremity. 4

(c) $f:\mathbb{R}\longrightarrow\mathbb{R}$ is so defined that $f(x)=\begin{cases}-1 & \text{when } x \text{ is rational}\\ +1 & \text{when } x \text{ is irrational}\end{cases}$ 4

Prove or disprove: f is continuous at no point in $\mathbb{R}$.

(d) Prove that any two left cosets of H in a group G have same number of elements. 2

4. (a) Prove that a monotonic sequence cannot have two subsequences one of which is convergent and the other is divergent. 5

(b) Find the coordinate of the centre of gravity of a figure bounded by the coordinate axes and the arc of $4a^2x^2+9a^2y^2=36$ situated in the first quadrant. 5

(c) Prove that every subgroup of a cyclic group is cyclic. 4

5. (a) Solve $x^3-12x^2-6x-10=0$ or $x^3-3a^2x-2a^3\cos 3A=0$ by Cardan's method. 4

(b) Define an accumulation point of a set $A\subseteq\mathbb{R}^2$ and an open set $A\subseteq\mathbb{R}^2$. Correct or justify: 4

$$S=\{(x,\, y)\,|\; 20<x^2+y^2<30\} \text{ is an open set in } \mathbb{R}^2.$$

(c) Prove that a square matrix is orthogonally diagonalisable if it is symmetric. Is the converse true? — Justify. 4

(d) Correct or justify the statement: Klein's 4-group is a cyclic group. 2

6. (a) If $a_1, a_2, \cdots\cdots, a_n$ and $b_1, b_2, \cdots\cdots, b_n$ are $2n$ real numbers, then prove that 3

$$(a_1b_1+a_2b_2+\cdots\cdots+a_nb_n)^2\le(a_1^2+a_2^2+\cdots\cdots+a_n^2)(b_1^2+b_2^2+\cdots\cdots+b_n^2)$$

(b) Prove that each eigen value of a real orthogonal matrix has unit modulus. 3

(c) State and prove Darboux theorem on derivative. 4

(d) Let G be the sets of all polynomials of the form ax^2+bx+c with coefficients from the set $\{1, 2, 3\}$. We can make G a group under addition by adding the polynomials in the usual way, except that we use modulo 3 to combine the coefficients. With this operation, prove that G is a group of order 27 that is not cyclic. 4

7. (a) Show that the transformation $x=r\cos\theta,\; y=r\sin\theta$ reduces the equation 5

$$xy\left(\frac{\partial^2u}{\partial x^2}-\frac{\partial^2u}{\partial y^2}\right)-(x^2-y^2)\frac{\partial^2u}{\partial x\partial y}=0 \text{ to } \left(r\frac{\partial^2u}{\partial r\partial\theta}-\frac{\partial u}{\partial\theta}\right)=0$$

(b) If $a, b, c\in\mathbb{R}$, show that the positive square root of $a^2+b^2+c^2-bc-ca-ab$ is greater than or equal to $\frac{\sqrt{3}}{2}\max\{|b-c|, |c-a|, |a-b|\}$. 3

(c) Examine convergence of $1-\frac{1}{3}\left(1+\frac{1}{3^2}\right)+\frac{1}{5}\left(1+\frac{1}{3^2}+\frac{1}{5^2}\right)-\cdots\cdots$. 3

(d) $u=(x_1, x_2, x_3)$, $v=(y_1, y_2, y_3)$ are any two elements of $\mathbb{R}^3$. A mapping $f:\mathbb{R}^3\times\mathbb{R}^3\longrightarrow\mathbb{R}$ is defined by $f(u, v)=x_1y_1+x_2y_2-x_3y_3$. Examine whether f is an inner product in $\mathbb{R}^3$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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