Mathematical Physics II

Numerical Methods using Python: Numerical Differentiation

In this topic we shall discuss how to get an approximate value of the first order derivative of a function $f(x)$. In this connection we shall cover **Forward** and **Backward Difference Formula** for Numerical Differentiation.

- **Forward Difference Formula:** From Taylor's Theorem we know that one can obtain the value of a function at a point $x + \Delta x$ if the function is known at x using the following relation:

$$f(x + \Delta x) = f(x) + \Delta x f'(x) + \frac{(\Delta x)^2}{2} f''(\theta);$$

where Δx is a small increment in x and $x < \theta < x + \Delta x$. Replacing Δx by h and rearranging the above relation one can get

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2} f''(\theta)$$

If h is sufficiently small then one can approximate

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

This is known as Forward Difference Formula. Here the neglected term $-\frac{h}{2} f''(\theta)$ is referred as truncation error E_t.

- **Backward Difference Formula:** Similarly if the increment in x, $\Delta x = -h$ then

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

This is known as Backward Difference Formula.

References

1. 'Numerical Methods', E. Balagurusamy, TMH, 2016.

- **Example 1**: In the following pictures sample python programs using forward and backward difference formulae have been shown to solve the following problem.

  Evaluate the first derivative at $x = -2.5$ and at $x = 2.5$ of the following table function:

x	-2.5	-1.5	-0.5	0.5	1.5	2.5
y	-23	-8	1.5	1	7.5	23.5

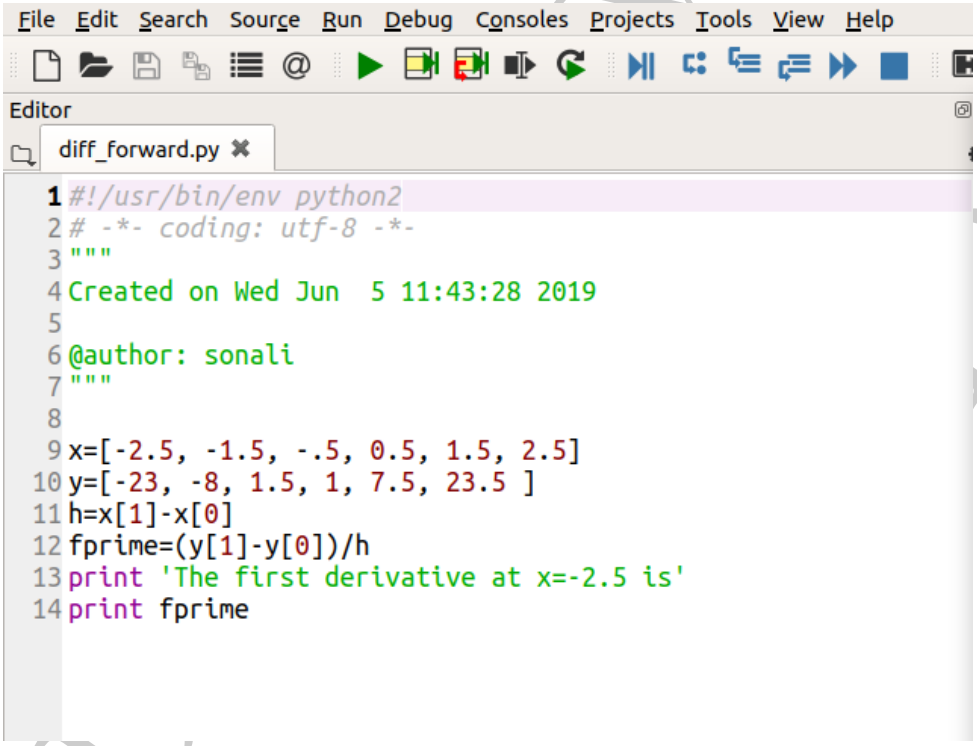

Figure 1: Sample Python Code for numerical differentiation using Forward Difference Formula

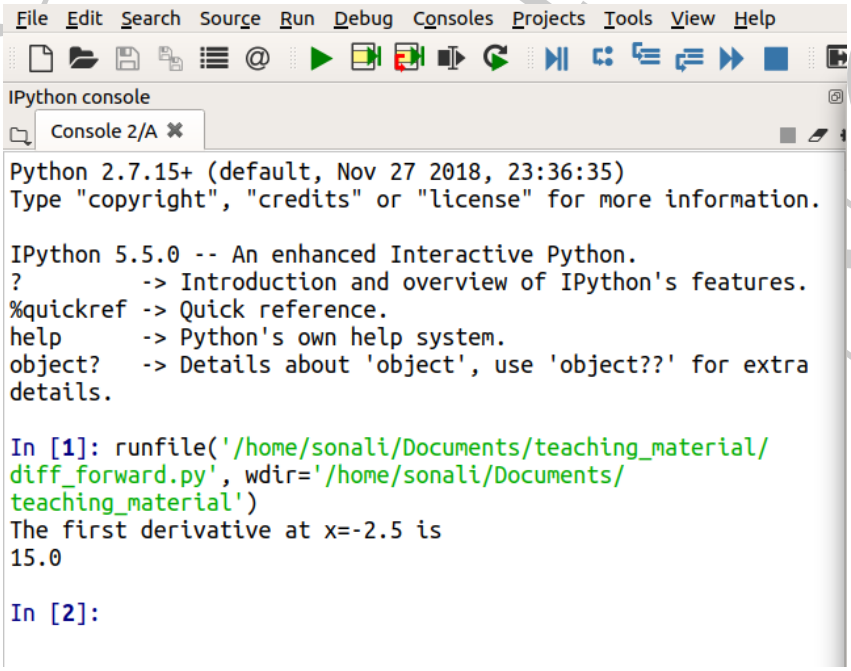

Figure 2: Output of the Python code using Forward Difference Formula

Now a sample code to determine the first derivative at $x = 2.5$ using backward difference formula follows as:

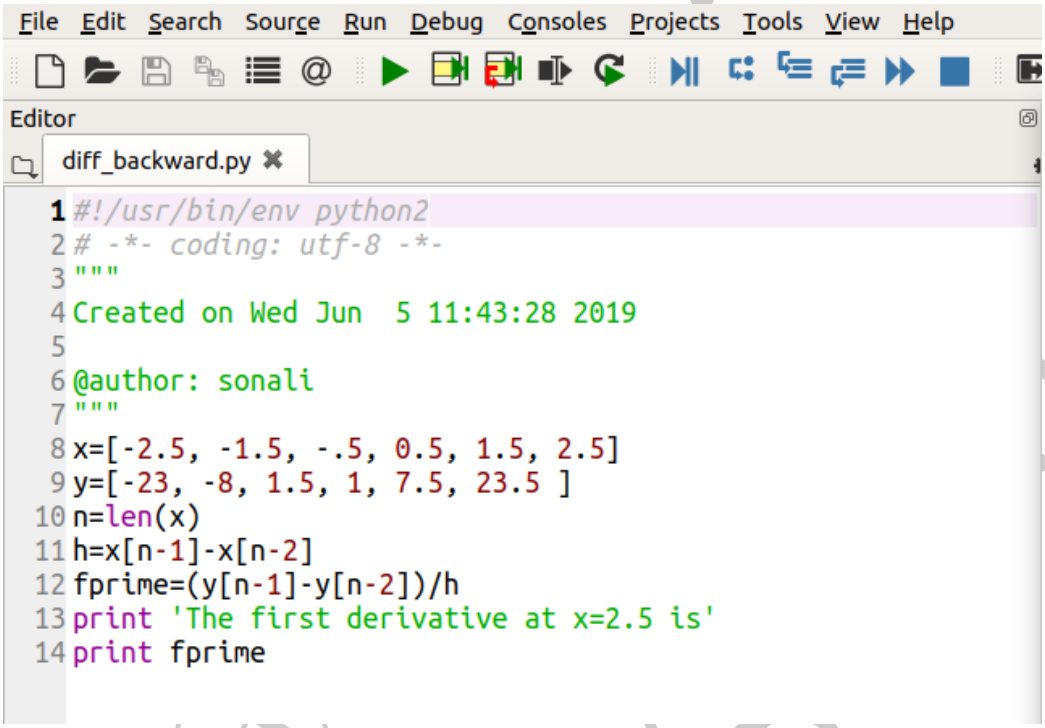

Figure 3: Sample Python Code for numerical differentiation using Backward Difference Formula

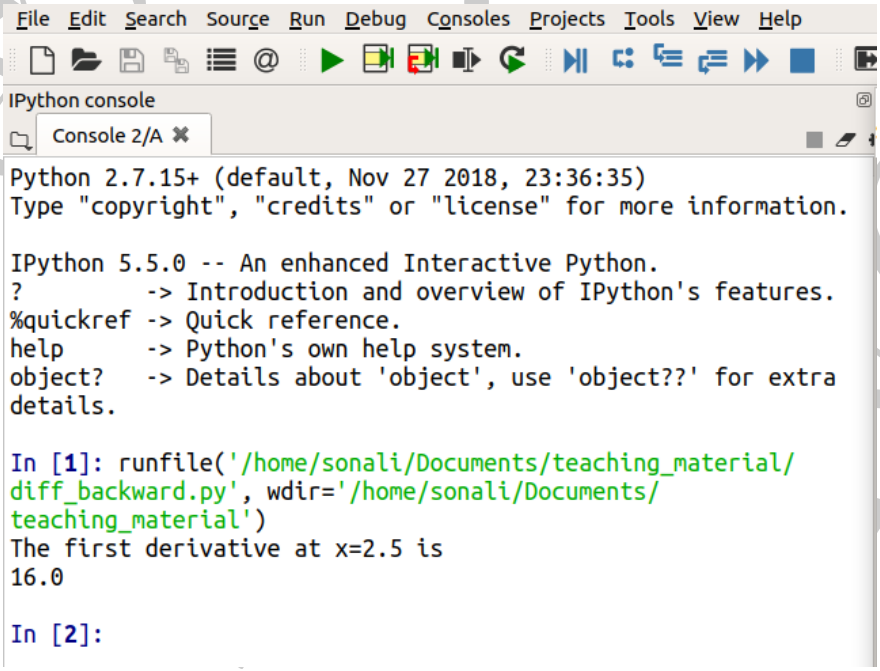

Figure 4: Output of the Python code using Backward Difference Formula

- **Example 2**: In the following pictures sample python program is shown to calculate first derivative of function $f(x) = x^2 - 3x + 1$ at a point $x = 1$ with a specified accuracy 10^{-5} using forward difference formula along with its output.

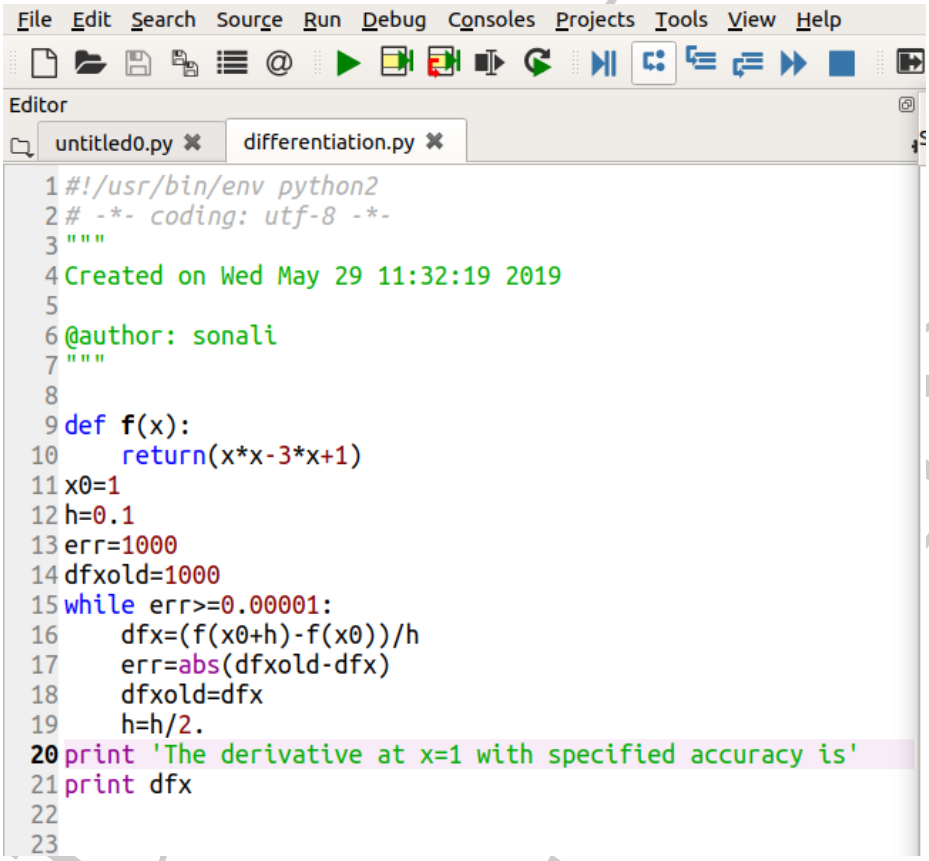

Figure 5: Sample Python Code for numerical differentiation of a function using Forward Difference Formula

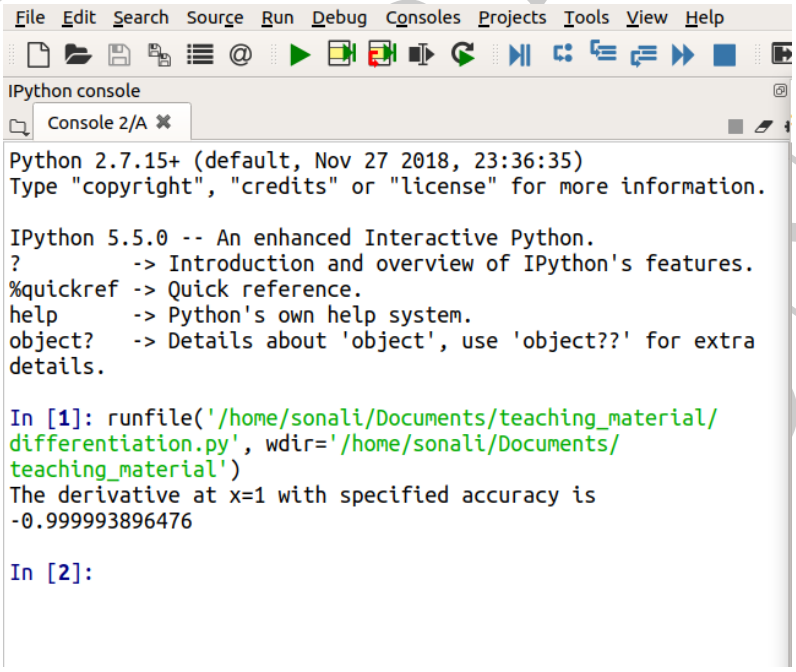

Figure 6: Output of the Python code using Forward Difference Formula