Equation of State of Spin Asymmetric QGP Matter

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Abstract

We calculate the free energy, entropy and pressure of the Quark Gluon Plasma (QGP) at finite temperature and density with a given fraction of spin-up and spin-down quarks using a MIT bag model. Within our phenomenological model, we estimate the transition temperature T_c by constructing the phase boundary between the hadronic phase and the QGP phase. Finally, we compute the equation of state of the QGP and its dependence on the temperature and the density.

Key words: QGP matter, thermodynamic properties, EOS.

1. Introduction

One of the active areas of high energy physics research is the study of thermodynamic properties of interacting hadronic matter in the extreme conditions of temperature and/or density for the last few decades. At such high temperature and/or density the hadrons are expected to dissolve into their more fundamental constituents *viz.* quarks and gluons, forming a new state of matter called Quark Gluon Plasma (QGP). The possibility of creating high temperature QGP by colliding heavy ions in the laboratory and studying this phase of matter has been the goal of experiments at CERN SPS and at the Relativistic Heavy Ion Collider (RHIC) facility at Brookhaven National Laboratory (BNL)[1;2;3]. ALICE, ATLAS and CMS Collaborations at the Large Hadron Collider (LHC) have provided further impetus to these studies[4;5;6]. This experimental search of the QGP needs reliable theoretical estimates of various signals which depend on the pressure, entropy, deconfinement temperature and the equation of state (EOS) etc.

In recent years, significant progress has been made to understand the behavior of QGP phase of matter leading to major advancement in the theoretical front addressing some of the subtle issues of the quasiparticles excitation in such an environment. For example, new lattice results for the equation of state of QCD with 2 + 1 dynamical flavors were obtained in[7;8]. Ref. 9 deals with hybrid model in constructing a deconfining phase boundary between the hadron gas and the QGP and provides a realistic EOS for the strongly interacting matter.

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In the present work, we calculate various thermodynamic quantities like energy density, pressure, entropy density within the one gluon exchange model for non-zero chemical potential and show how these quantities can be expressed in terms of the spin polarization parameter ξ and the temperature T. Within our model, we estimate the critical temperature T_c for the transition between the hadronic phase to the quark phase. Furthermore, we determine the equation of state of QGP and its dependence on the temperature and the density.

2. Thermodynamic properties

In this section we calculate the thermodynamic properties, like energy density (E), pressure ($\mathcal{P}$), entropy density ($\mathcal{S}$), free energy ($\mathcal{E}$), for QGP matter with explicit spin dependent quarks by using a MIT bag model with fixed bag pressure $\mathcal{B}$. The bag constant ($\mathcal{B}$) is the difference between the energy densities of non-interacting and interacting quarks. Within this model, for the calculation of energy density and other related quantities, we assume the QGP is composed of the light quarks only, i.e. the up and down quarks which interact weakly, and the gluons which are treated as almost free[10]. We consider the color symmetric forward scattering amplitude of two quarks around the Fermi surface by the one gluon exchange interaction. The direct term does not contribute as it involves the trace of single color matrices, which vanishes, while the leading contribution comes from the exchange term[11]. The quasiparticle interaction ($f^{ss'}_{pp'}$) can be decomposed into two parts, spin may be either parallel ($s = s'$) or antiparallel ($s = -s'$) corresponding to spin nonflip ($f^{\rm nf}_{pp'}$) or spin flip ($f^{\rm f}_{pp'}$) scattering amplitudes[11;12], such that

$$f^{ss'}_{pp'} \;=\; f^{\rm nf}_{pp'} + \frac{1}{2} f^{\rm f}_{pp'}, \tag{1}$$

where p and p' are the momentum of the quasiparticles. Here, the factor $1/2$ is due to the equal scattering possibilities involving spin-up spin-down and spin-down spin-up quarks.

The leading contributions to the energy density of quarks are given by two terms viz. kinetic, exchange energy densities, i.e.

$$E_q \;=\; E_{kin} + E_{ex}. \tag{2}$$

The total kinetic energy density for spin-up and spin-down quarks, including the color and flavor degeneracy factors N_c and N_f for quarks, is[13]

$$\begin{aligned} E_{kin} &= N_c N_f \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3} \varepsilon_p n^s_p(T) \\ &= \frac{3}{(2\pi)^2} \Big\{ p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3}] \\ &\quad + 2\pi^2 T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}] + \frac{14}{15}\pi^4 T^4 \Big\}. \end{aligned} \tag{3}$$

Here, ε_p and $n^s_p(T)$ are the single particle energy and the Fermi distribution function respectively. $\xi = (n^+_q - n^-_q)/(n^+_q + n^-_q)$ is the spin polarization parameter with $0 \le \xi \le 1$. n^+_q and n^-_q correspond to the densities of spin-up and spin-down quarks, respectively and p_f is the Fermi momentum of the unpolarized matter ($\xi = 0$).

For spin asymmetric quarks, the exchange energy density consists of two terms $E_{ex} = E_{ex}^{\rm nf} + E_{ex}^{\rm f}$, and can be determined by evaluating the following integrals[11;12;13]

$$E_{ex}^{\rm nf} = \frac{N_f N_c^2}{2}\sum_{s=\pm}\int\int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} f_{pp'}^{\rm nf}\; n_p^s(T)\; n_{p'}^s(T), \tag{4}$$

$$E_{ex}^{\rm f} = N_f N_c^2 \int\int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} f_{pp'}^{\rm f}\; n_p^s(T)\; n_{p'}^s(T). \tag{5}$$

The analytical expression for the total exchange energy density is found to be[13]

$$\begin{aligned} E_{ex} &= \frac{g^2}{(2\pi)^4}\Big\{p_f^4[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] \\ &+\frac{4}{3}\pi^2T^2p_f^2[(1+\xi)^{2/3} + (1-\xi)^{2/3}] + \frac{4}{9}\pi^4T^4\Big\}. \end{aligned} \tag{6}$$

For the energy density of gluons we have

$$\begin{aligned} E_g &= 16\int \frac{d^3k}{(2\pi)^3}\frac{k}{e^{k/T}-1} \\ &= \frac{8}{15}\pi^2T^4, \end{aligned} \tag{7}$$

where 16 is the degeneracy of gluons[10;13].

Since our system is ultra-relativistic, there is a simple relation between the pressure and the energy density:

$$\mathcal{P} = \frac{1}{3}E. \tag{8}$$

To obtain the total energy density of the QGP, the bag pressure ($\mathcal{B}$) should be included in addition to the quark and gluon contributions[10;13],

$$E_{QGP} = E_q + E_g + \mathcal{B}. \tag{9}$$

The pressure of the system is determined to be

$$\mathcal{P}_{QGP} = \mathcal{P}_q + \mathcal{P}_g - \mathcal{B}. \tag{10}$$

Once the value of $\mathcal{P}$ is determined, one can readily calculate the entropy density of the system by evaluating

$$\mathcal{S}_{QGP} = \frac{\partial}{\partial T}(\mathcal{P}_{QGP}). \tag{11}$$

The thermodynamic properties of the system can be obtained by using the Helmholtz free energy relation

$$\mathcal{E}_{QGP} = E_{QGP} - T\mathcal{S}_{QGP}. \tag{12}$$

For the numerical estimation of all these quantities, we take $\alpha_c = \frac{g^2}{4\pi} = 0.2$, as the coupling constant of QCD and the bag pressure $\mathcal{B} = 208$ MeV fm^{-3} for zero hadronic pressure[10;13].

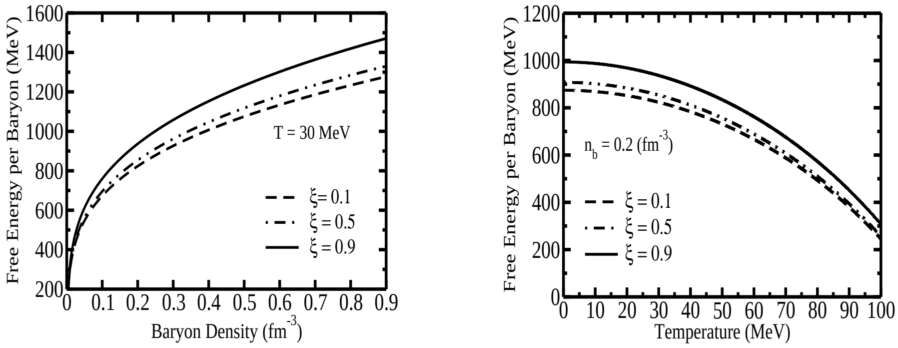

Figure 1: Density dependence of the free energy per baryon (left panel) and Temperature dependence of the free energy per baryon (right panel) for various ξ.

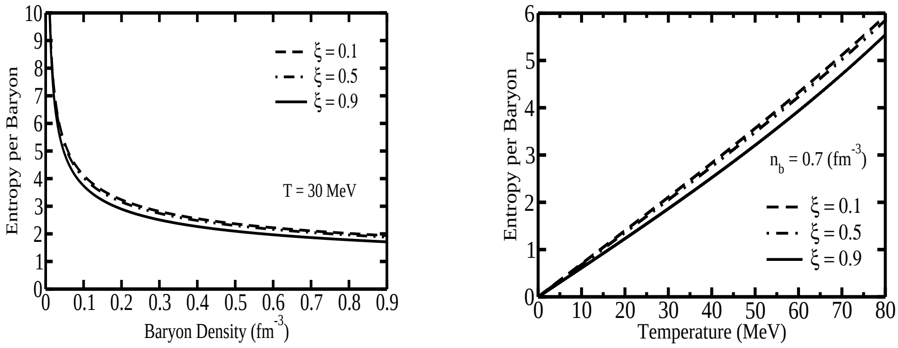

Figure 2: Density dependence of the entropy per baryon (left panel) and Temperature dependence of the entropy per baryon (right panel) for various ξ.

In the left panel of the Fig. 1, it has been observed that the free energy is larger with higher value of ξ and in the right panel it is shown that the free energy decreases with increasing temperature. Therefore QGP is more stable when quarks are unpolarized.

In the left panel of Fig. 2, it has been observed that the entropy decreases with increasing baryon density and at low density, entropy is approximately the same for all the values of the order parameter at a fixed temperature. From the right panel it has been observed, entropy per baryon in the QGP is an increasing function of temperature and that the entropy decreases with increasing ξ. The numerical estimates suggest that the entropy per baryon is continuous along the phase boundary indicating a cross over from hadronic phase to QGP phase.

Similarly, in the left panel of Fig. 3, it is shown that at the same baryon density the pressure is larger for the interacting cases in comparison with the non-interacting one and if the interaction strength increases also the pressure raises further. Thus the interaction makes easier the quarks transition to the deconfined phase at lower density. Therefore, the interaction of the QCD coupling constant reduces the value of the density to reach a transition. In the right panel of Fig. 3, it has been observed that at a constant temperature, the pressure of QGP is larger for polarized quarks than for unpolarized ones. Thus the equation of state of spin asymmetric QGP matter becomes stiffer by increasing

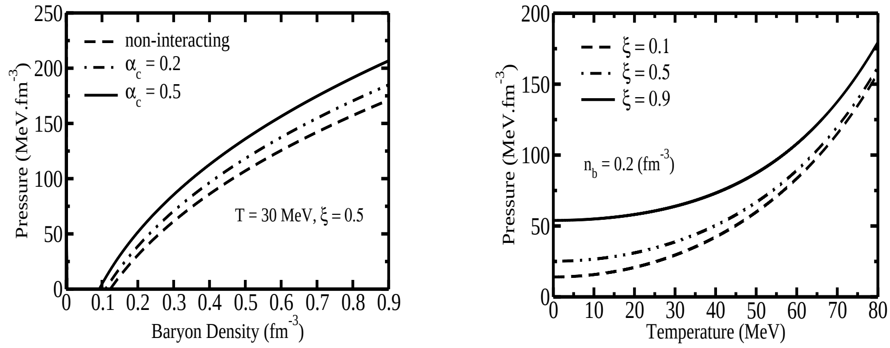

Figure 3: The pressure for different coupling constants as a function of baryon density (left panel) and the pressure as a function of temperature for various ξ (right panel).

the baryon density (temperature)[10;13].

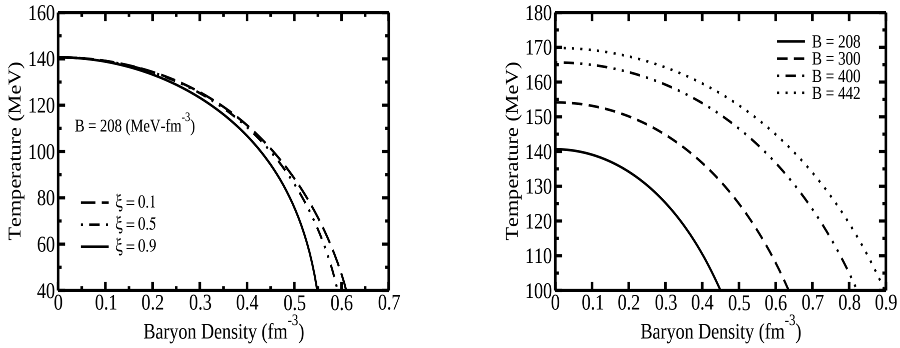

Figure 4: The phase diagram with different ξ (left panel) and with different bag pressure (right panel).

The QGP phase diagrams are depicted in Fig. 4; in the left panel, we see that the critical temperature is independent of the polarization parameter while critical density is different for unpolarized and polarized QGP. On the other hand, in the right panel it is shown that the critical temperature and density for the phase transition are different for different bag pressures and the critical values increase when increasing the bag pressure. It is shown that the critical temperature for the deconfined phase transition lies between $130 < T_c < 170$ MeV[13].

3. Summary and Discussions

In this work, we have investigated the thermodynamic properties of the QGP composed of the massless spin-up and spin-down quarks at finite temperature and density using a MIT bag model within one gluon exchange (OGE) interaction. It was shown that the free energy increases by increasing the polarization parameter. This fact may suggests that the QGP with unpolarized quark is energetically favorable. We found that the entropy is a decreasing function of the density and an increasing function of the temperature. In the present phenomenological model, the equation of state (EOS) of the QGP with different order parameters has been computed. It was shown that the increase of temperature or density makes the EOS stiffer. We obtained a phase boundary for quark-hadron phase transition by making the bag constant independent of μ and T. The values of critical parameters have been estimated for the transition from the hadronic phase to the QGP phase by using the EOS. It has been observed that the entropy per baryon is continuous along the phase boundary, indicating a cross over from the hadronic phase to the QGP phase.

It has to be mentioned that our results depend on the values of the bag pressure while a change in the value of the QCD coupling does not have any dramatic effect in our calculations. Within the scope of the present model, the value of T_c obtained here, is close to the lattice QCD prediction.

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