Review on Boundary Conditions and Noncommutativity in bosonic String Theory

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Abstract

We start with the gauge independent analysis of Polyakov string and give a review of the emergence of noncommutativity in the context of an open string. The boundary conditions are not treated as constraints, but they are systematically implemented by modifying the canonical Poisson bracket structure.

Keywords: Strings, Noncommutativity

1. Introduction

An intriguing connection between string theory, noncommutative geometry and noncommutative Yang-Mills theory was revealed in[1]. The study of open string, in the presence of a background Neveu-Schwarz two-form field $B_{\mu\nu}$, leads to a noncommutative(NC) structure which manifests in the noncommutativity at the end points of the string which are attached to D-branes. Different approaches have been adopted to obtain this result.

We discuss the Polyakov action and also the essential result of[2;3;4] in which the authors provide an exhaustive analysis of the noncommutativity in open string theory moving in the presence of a constant Neveu-Schwarz field, in the conventional Hamiltonian framework. In contrast to the usual studies, this model of string theory is very general in the sense that no gauge is fixed at the beginning. Let us recall that all computations of noncommutativity, mentioned before, were done in the conformal gauge. This gauge independent analysis yields a new noncommutative structure, which correctly reduces to the usual one in conformal gauge. This shows the compatibility of the present analysis with the existing literature. In the general case, the noncommutativity is manifested at all points of the string, in contrast to conformal gauge results where it appears only at the boundaries. Indeed, in this gauge independent scheme, one finds a noncommutative algebra among the coordinates, even for a free string, a fact that was not observed before. Expectedly, this noncommutativity vanishes in the conformal gauge. Note however, that there is no gauge for which noncommutativity vanishes in the interacting theory.

At the outset, let us point out the crucial difference between existing Hamiltonian analysis[5] and this approach. This is precisely in the interpretation of the boundary conditions(BC) arising in the string theory. The general consensus has been to consider the boundary conditions as primary constraints of the theory and attempt a conventional Dirac constraint analysis[6]. The aim is to induce the noncommutativity in the form of Dirac Brackets between coordinates. The subsequent analysis turns out to be ambiguous since it involves the presence of $\delta(0)$-like factors, (see Chu and Ho in[5]). Different results are obtained depending on the interpretation of these factors.

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Here on the other hand we do not treat the BCs as constraints, but show that they can be systematically implemented by modifying the canonical Poisson Bracket(PB) structure. In this sense this approach is quite similar in spirit to that of Hanson, Regge and Teitelboim[7], where modified PBs were obtained for the free Nambu-Goto string, in the orthonormal gauge, which is the counterpart of the conformal gauge in the free Polyakov string.

2. The free string in Polyakov formalism

In this section, we analyze the Polyakov formulation of the free string. The Polyakov action for a free bosonic string reads,

$$S_P = -\frac{1}{2}\int_{-\infty}^{+\infty} d\tau \int_0^{\pi} d\sigma \, \sqrt{-g} g^{ab} \partial_a X^\mu \, \partial_b X_\mu \tag{1}$$

where τ and σ are the usual world-sheet parameters and g_{ab}, up to a Weyl factor, is the induced metric on the world-sheet. $X^\mu(\sigma)$ are the string coordinates in the D-dimensional Minkowskian target space with metric $\eta_{\mu\nu} = \text{diag}\ (-1, 1, 1.., 1)$.
This action has the following symmetries:

- 1. D-dimensional Poincaré invariance:

$$\begin{aligned} X'^\mu(\tau,\sigma) &= \Lambda^\mu{}_\nu X^\nu(\tau,\sigma) + a^\mu \\ g'_{ab}(\tau,\sigma) &= g_{ab}(\tau,\sigma) \end{aligned} \tag{2}$$

- 2. Diffeomorphism Invariance:

$$\begin{aligned} X'^\mu(\tau',\sigma') &= X^\mu(\tau,\sigma) \\ \frac{\partial \sigma'^c}{\partial \sigma^a}\frac{\partial \sigma'^d}{\partial \sigma^b} g'_{cd}(\tau',\sigma') &= g_{ab}(\tau,\sigma) \end{aligned} \tag{3}$$

  for new coordinates $\sigma'^a(\tau,\sigma)$.

- 3. Two-dimensional Weyl invariance:

$$\begin{aligned} X'^\mu(\tau,\sigma) &= X^\mu(\tau,\sigma) \\ g'_{ab}(\tau,\sigma) &= \exp(2\omega(\tau,\sigma))\, g_{ab}(\tau,\sigma) \end{aligned} \tag{4}$$

  for arbitrary $\omega(\tau,\sigma)$.

Here we carry out our analysis in the complete space by regarding both X^μ and g_{ab} as independent dynamical variables[8]. The canonical momenta are,

$$\begin{aligned} \Pi_\mu &= \frac{\delta \mathcal{L}_P}{\delta\left(\partial_\tau X^\mu\right)} = -\sqrt{-g}\,\partial_\tau X_\mu \\ \pi_{ab} &= \frac{\delta \mathcal{L}_P}{\delta(\partial_\tau g^{ab})} = 0. \end{aligned} \tag{5}$$

It is clear that while Π_μ is a genuine momenta, $\pi_{ab} \approx 0$ are the primary constraints of the theory. The conservation of the above primary constraints leads to the secondary constraints $\Omega_1(\sigma)$ and $\Omega_2(\sigma)$. These secondary constraints also follow from the equation obtained by varying g_{ab} since this is basically a Lagrange multiplier. This imposes the vanishing of the symmetric energy-momentum tensor,

$$T_{ab} = \frac{2}{\sqrt{-g}}\frac{\delta S_P}{\delta g^{ab}} = -\partial_a X^\mu \, \partial_b X_\mu + \frac{1}{2} g_{ab} g^{cd} \partial_c X^\mu \, \partial_d X_\mu = 0. \tag{6}$$

Because of the Weyl invariance, the energy-momentum tensor is traceless,

$$T^a{}_a = g^{ab} T_{ab} = 0$$

so that only two components of T_{ab} are independent. These components, which are the constraints of the theory, are given by,

$$\begin{aligned} \Omega_1(\sigma) &= g\,T^{00} = -T_{11} = \left(\Pi^2(\sigma) + X'^2(\sigma)\right) = 0 \\ \Omega_2(\sigma) &= \sqrt{-g}T^0{}_1 = \Pi(\sigma) \cdot X'(\sigma) = 0 \end{aligned} \tag{7}$$

The canonical Hamiltonian obtained from (1) by a Legendre transformation is given by,

$$H = \int d\sigma\,\sqrt{-g}T^0{}_0 = \int d\sigma \sqrt{-g}\left(\frac{1}{2g_{11}}\,\Omega_1(\sigma) + \frac{g_{01}}{\sqrt{-g}\,g_{11}}\,\Omega_2(\sigma)\right) \tag{8}$$

expectedly, the Hamiltonian turns out to be a linear combination of the constraints.

Just as variation of g_{ab} yields the constraints, variation of X^μ gives the equation of motion,

$$\partial_a\left(\sqrt{-g}g^{ab}\,\partial_b X^\mu\right) = 0 \tag{9}$$

Finally, there is a mixed BC,

$$\partial^\sigma\, X^\mu(\tau, \sigma)|_{\sigma=0,\pi} = 0 \tag{10}$$

In the covariant form involving phase space variables, this is given by

$$\left(\partial_\sigma\, X^\mu + \sqrt{-g}g^{01}\,\Pi^\mu\right)|_{\sigma=0,\pi} = 0. \tag{11}$$

The non trivial basic PBs of the theory are:

$$\begin{aligned} \left\{X^\mu(\tau,\sigma), \Pi_\nu(\tau,\sigma')\right\} &= \delta^\mu_\nu\,\delta(\sigma-\sigma') \\ \left\{g_{ab}(\tau,\sigma), \pi^{cd}(\tau,\sigma')\right\} &= \frac{1}{2}(\delta^c{}_a\delta^d{}_b + \delta^d{}_a\delta^c{}_b)\delta(\sigma-\sigma') \end{aligned} \tag{12}$$

where $\delta(\sigma - \sigma')$ is the usual one-dimensional Dirac delta function. From the basic PB, it is easy to generate the following first class (involutive) algebra,

$$\begin{aligned} \{\Omega_1(\sigma), \Omega_1(\sigma')\} &= 4\,(\Omega_2(\sigma) + \Omega_2(\sigma'))\partial_\sigma\delta(\sigma-\sigma'), \\ \{\Omega_2(\sigma), \Omega_1(\sigma')\} &= (\Omega_1(\sigma) + \Omega_1(\sigma'))\partial_\sigma\delta(\sigma-\sigma'), \\ \{\Omega_2(\sigma), \Omega_2(\sigma')\} &= (\Omega_2(\sigma) + \Omega_2(\sigma'))\partial_\sigma\delta(\sigma-\sigma'). \end{aligned} \tag{13}$$

3. Modified brackets for the Polyakov string

Let us again consider the BCs for the Polyakov string,

$$\left(\partial_\sigma\, X^\mu + \sqrt{-g}g^{01}\,\Pi^\mu\right)|_{\sigma=0,\pi} = 0. \tag{14}$$

It is easily seen that the above BCs are incompatible with the first of the basic PBs (12). Hence the brackets should be modified suitably. The modification of PBs can be done in spirit to the treatment of Hanson *et al.*[7], where modified PBs were obtained for the free Nambu-Goto string.

We would also like to mention that there is an apparent contradiction of the constraint $\pi_{ab} \approx 0$ with the second PB (12). However this equality is valid in Dirac's "weak" sense only, so that it can be set equal to zero only after the relevant brackets have been computed. These weak equalities will be designated by $\approx$, rather than an equality, which is reserved only for a strong equality. In this sense, therefore, there is no clash between this constraint and the relevant PB. Indeed, we can even ignore the canonical pair (g_{ab}, π^{cd}) from the basic PB.

The situation is quite similar to usual electrodynamics. There the Lagrange multiplier is A_0, which corresponds to g_{ab} in the string theory. The multiplier A_0 enforces the Gauss constraint just as g_{ab} enforces the constraints Ω_1 and Ω_2. Furthermore, the Gauss constraint generates the time independent gauge transformations, while Ω_1, Ω_2 generate the diffeomorphism transformations.

The BC (14), on the other hand, is not a constraint in the Dirac sense[6], since it is applicable only at the boundary. Thus, there has to be an appropriate modification in the PB, to incorporate this condition. This is not unexpected and occurs, for instance, in the example of a free scalar field $\phi(x)$ in (1+1) dimension, subjected to periodic BC of period, say, 2π ($\phi(t, x+2\pi) = \phi(t, x)$). There the PB between the field $\phi(t, x)$ and its conjugate momentum $\pi(t, x)$ are given by,

$$\{\phi(t, x), \pi(t, y)\} = \delta_P(x - y) \tag{15}$$

where,

$$\delta_P(x - y) = \delta_P(x - y + 2\pi) = \frac{1}{2\pi} \sum_{n \in \mathcal{Z}} e^{in(x-y)} \tag{16}$$

is the periodic delta function of period 2π[9] and occurs in the closure properties of the basis functions e^{inx} for the space of square integrable functions, defined on the unit circle S^1. This periodic delta function is related to the usual Dirac delta function as $\delta_P(x - y) = \sum_{n \in \mathcal{Z}} \delta(x - y + 2\pi n)$

Before discussing the mixed type condition (14), that emerged in a completely gauge independent formulation of the Polyakov action, consider the simpler Neumann type condition $(\partial_\sigma X^\mu)|_{\sigma=0,\pi} = 0$ in an orthonormal (conformal) gauge. It is easy to find the solutions to the equations of motion (9) which are compatible with the Neumann BCs

$$X^\mu(\tau, \sigma) = x^\mu + p^\mu \tau + i \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-in\tau} \cos(n\sigma) \tag{17}$$

Reality of $X^\mu(\tau, \sigma)$ implies that x, p are real and

$$\alpha_n^{\mu\star} = \alpha_{-n}^\mu \quad \text{for } n \neq 0. \tag{18}$$

We enlarge the domain of definition of the bosonic field X^μ from $[0, \pi]$ to $[-\pi, \pi]$ by observing the fact

$$X^\mu(\tau, -\sigma) = X^\mu(\tau, \sigma) \text{ under } \sigma \to -\sigma \tag{19}$$

which further yields

$$X^\mu(-\pi) = X^\mu(\pi). \tag{20}$$

Now we start by noting that the usual properties of a delta function is also satisfied by $\delta_P(x)$ (16),

$$\int_{-\pi}^{\pi} dx' \, \delta_P(x' - x) \, f(x') = f(x) \tag{21}$$

for any periodic function $f(x) = f(x + 2\pi)$ defined in the interval $[-\pi, \pi]$. Then by using (19), the above integral (21) reduces to the following:

$$\int_0^\pi d\sigma' \, \Delta_+(\sigma', \sigma) \, X^\mu(\sigma') = X^\mu(\sigma) \tag{22}$$

where

$$\Delta_+(\sigma', \sigma) = \delta_P(\sigma' - \sigma) + \delta_P(\sigma' + \sigma). \tag{23}$$

Using (16), the explicit form of $\Delta_+(\sigma',\sigma)$ can be given as,

$$\Delta_+(\sigma',\sigma) = \frac{1}{\pi} + \frac{1}{\pi}\sum_{n\neq 0}\cos(n\sigma')\cos(n\sigma). \tag{24}$$

It thus follows that the appropriate PB is given by,

$$\left\{X^\mu(\tau,\sigma),\Pi_\nu(\tau,\sigma')\right\} = \delta^\mu_\nu\Delta_+(\sigma',\sigma). \tag{25}$$

It is clearly consistent with Neumann BC as $\partial_\sigma\,\Delta_+(\sigma,\sigma')|_{\sigma=0,\pi} = \partial_{\sigma'}\,\Delta_+(\sigma,\sigma')|_{\sigma=0,\pi} = 0$ and is automatically satisfied. Observe also that the other brackets

$$\left\{X^\mu(\tau,\sigma),X^\nu(\tau,\sigma')\right\} = 0 \tag{26}$$
$$\{\Pi^\mu(\tau,\sigma),\Pi^\nu(\tau,\sigma)\} = 0 \tag{27}$$

are already consistent with the Neumann BCs and hence remain unchanged.

For a gauge independent analysis, we take recourse to the mixed condition (14). A simple inspection shows that this is also compatible with the modified brackets (25 , 27), but not with (26). Hence the bracket among the coordinates should be altered suitably. We therefore make an ansatz,

$$\{X^\mu(\tau,\sigma),X^\nu(\tau,\sigma')\} = C^{\mu\nu}(\sigma,\sigma') \tag{28}$$

where,

$$C^{\mu\nu}(\sigma,\sigma') = -C^{\nu\mu}(\sigma',\sigma).$$

Imposing the BC (14) on this algebra, we get,

$$\begin{aligned}\partial_{\sigma'}\,C^{\mu\nu}(\sigma,\sigma')|_{\sigma'=0,\pi} &= \partial_\sigma\,C^{\mu\nu}(\sigma,\sigma')|_{\sigma=0,\pi}\\ &= -\sqrt{-g}g^{01}\{\Pi^\mu(\tau,\sigma),X^\nu(\tau,\sigma')\}\\ &= \sqrt{-g}g^{01}\eta^{\mu\nu}\Delta_+(\sigma,\sigma')\end{aligned} \tag{29}$$

For an arbitrary form of the metric tensor, it might be technically problematic to find a solution for $C^{\mu\nu}(\sigma,\sigma')$. However, for a restricted class of metric[1] that satisfy

$$\partial_\sigma g_{ab} = 0$$

it is possible to give a quick solution of $C^{\mu\nu}(\sigma,\sigma')$ as,

$$C^{\mu\nu}(\sigma,\sigma') = \sqrt{-g}g^{01}\eta^{\mu\nu}\left[\Theta(\sigma,\sigma') - \Theta(\sigma',\sigma)\right] \tag{30}$$

where the generalised step function $\Theta(\sigma,\sigma')$ satisfies,

$$\partial_\sigma\Theta(\sigma,\sigma') = \Delta_+(\sigma,\sigma') \tag{31}$$

An explicit form of Θ is given by[7],

$$\Theta(\sigma,\sigma') = \frac{\sigma}{\pi} + \frac{1}{\pi}\sum_{n\neq 0}\frac{1}{n}\sin(n\sigma)\cos(n\sigma')\ , \tag{32}$$

having the properties,

$$\begin{aligned}\Theta(\sigma,\sigma') &= 1 \quad \text{for} \quad \sigma > \sigma'\ ,\\ \Theta(\sigma,\sigma') &= 0 \quad \text{for} \quad \sigma < \sigma'.\end{aligned} \tag{33}$$

[1]Such conditions also follow from a standard treatment of the light-cone gauge[10]

Using these relations, the simplified structure of noncommutative algebra follows,

$$\begin{aligned}\{X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')\} &= 0 \quad \text{for} \quad \sigma=\sigma' \\ \{X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')\} &= \pm\sqrt{-g}\, g^{01}\, \eta^{\mu\nu} \quad \text{for} \quad \sigma\neq\sigma' \end{aligned} \tag{34}$$

respectively. Thus a noncommutative algebra for distinct coordinates $\sigma\neq\sigma'$ of the string emerges automatically in a free string theory if a gauge independent analysis is carried out like this. But this non-commutativity can be made to vanish in gauges like conformal gauge, where $g^{01}=0$, thereby restoring the usual commutative structure.

Now using the modified basic brackets we obtain the following involutive constraint algebra

$$\begin{aligned}\{\Omega_1(\sigma), \Omega_1(\sigma')\} &= \Omega_1(\sigma')\partial_\sigma\Delta_+\left(\sigma,\sigma'\right)+\Omega_1(\sigma)\partial_\sigma\Delta_-\left(\sigma,\sigma'\right) \\ \{\Omega_1(\sigma), \Omega_2(\sigma')\} &= \left(\Omega_2(\sigma)+\Omega_2(\sigma')\right)\partial_\sigma\Delta_+\left(\sigma,\sigma'\right) \\ \{\Omega_2(\sigma), \Omega_2(\sigma')\} &= 4\left(\Omega_1(\sigma)\partial_\sigma\Delta_+\left(\sigma,\sigma'\right)+\Omega_1(\sigma')\partial_\sigma\Delta_-\left(\sigma,\sigma'\right)\right). \end{aligned} \tag{35}$$

A crucial intermediate step in the above derivation is to use the relation,

$$\{X'^\mu(\sigma), X'^\nu(\sigma')\} = 0 \tag{36}$$

which follows from the basic bracket (34)[2].

4. Interacting Polyakov string

The Polyakov action for a bosonic string moving in the presence of a constant background Neveu-Schwarz two-form field $B_{\mu\nu}$ is given by,

$$S_P = -\frac{1}{2}\int d\tau d\sigma\left(\sqrt{-g}g^{ab}\,\partial_a X^\mu\,\partial_b X_\mu + e\epsilon^{ab}\,B_{\mu\nu}\,\partial_a X^\mu\,\partial_b X^\nu\right) \tag{37}$$

where $\epsilon^{01}=-\epsilon^{10}=+1$. A usual canonical analysis leads to the following set of primary first class constraints,

$$\begin{aligned} gT^{00} &= \frac{1}{2}\left[(\Pi_\mu + eB_{\mu\nu}\partial_\sigma X^\nu)^2 + (\partial_\sigma X)^2\right]\approx 0 \\ \sqrt{-g}\,T^0_{\ 1} &= \Pi.\partial_\sigma X \approx 0 \end{aligned} \tag{38}$$

where

$$\Pi_\mu = -\sqrt{-g}\partial^\tau X_\mu + eB_{\mu\nu}\,\partial_\sigma X^\nu \tag{39}$$

is the momentum conjugate to X^μ. The boundary condition written in terms of phase-space variables is

$$\left[\partial_\sigma X_\mu + \Pi^\rho\left(NM^{-1}\right)_{\rho\mu}\right]_{\sigma=0,\pi} = 0 \tag{40}$$

where,

$$\begin{aligned} M^\rho{}_\mu &= \frac{1}{g_{11}}[\delta^\rho{}_\mu - \frac{2e}{\sqrt{-g}}\,g_{01}B^\rho{}_\mu + e^2\,B^{\rho\nu}B_{\nu\mu}] \\ N_{\nu\mu} &= -\frac{g^{01}}{g^{00}\sqrt{-g}}\,\eta_{\nu\mu} - \frac{1}{g_{11}}\,e\,B_{\nu\mu} \end{aligned} \tag{41}$$

are two matrices.

The $\{X^\mu, \Pi_\nu\}$ Poisson bracket is the same as that of the free string whereas considering the general structure (28) and exploiting the above boundary condition, one obtains

$$\partial_\sigma C_{\mu\nu}(\sigma,\sigma')\mid_{\sigma=0,\pi} = \left(NM^{-1}\right)_{\nu\mu}\Delta_+(\sigma,\sigma')\mid_{\sigma=0,\pi}. \tag{42}$$

As in the free case, we restrict to the class of metrics defined satisfying $\partial_\sigma g_{ab} = 0$, the above equation has a solution

$$\begin{aligned} C_{\mu\nu}(\sigma,\sigma') &= \frac{1}{2}(NM^{-1})_{(\nu\mu)}\left[\Theta(\sigma,\sigma') - \Theta(\sigma',\sigma)\right] \\ &+\frac{1}{2}(NM^{-1})_{[\nu\mu]}\left[\Theta(\sigma,\sigma') + \Theta(\sigma',\sigma) - 1\right]. \end{aligned} \quad (43)$$

where $(NM^{-1})_{(\nu\mu)}$ the symmetric and $(NM^{-1})_{[\nu\mu]}$ the antisymmetric part of $(NM^{-1})_{\nu\mu}$. The modified algebra is gauge dependent; it depends on the choice of the metric. However, there is no choice, for which the non-commutativity vanishes. To show this, note that the origin of the non-commutativity is the presence of non-vanishing $N^{\nu\mu}$ in the BC (40). Vanishing $N^{\nu\mu}$ would make $B_{\mu\nu}$ and $\eta_{\mu\nu}$ proportional to each other which obviously cannot happen, as the former is an antisymmetric and the latter is a symmetric tensor. Hence non-commutativity will persist for any choice of world-sheet metric g_{ab}.

5. Summary

Here we have discussed the Polyakov string and derived the expressions for a noncommutative algebra, that are more general than the standard results found in the conformal gauge. The origin of any modification in the usual Poisson algebra is the presence of boundary conditions. This phenomenon is quite well known for a free scalar field subjected to periodic boundary conditions. We showed that its exact analogue is the conformal gauge fixed free string, where the boundary condition is of Neuman-type. This led to a modification only in the $\{X^\mu(\sigma), \Pi_\nu(\sigma')\}$ algebra, where the usual Dirac delta function got replaced by $\Delta_+(\sigma,\sigma')$. Using certain algebraic consistency requirements, we showed that the boundary conditions in the free theory naturally led to a noncommutative structure among the coordinates. This non-commutativity, however, vanishes in the conformal gauge, as expected. The same technique was adopted for the interacting string. Here, on contrast, we find that there is a genuine noncommutativity at the string end points and can not be made to vanish in any gauge.

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