Gravitational Instability in the Presence of Primordial Fully Tangled Magnetic Fields

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Abstract

Primordial magnetic fields can provide the requisite initial conditions for the generation of additional density perturbations. The role of these fields, in the context of structure formation, however has to be investigated by studying the conditions for collapse of a large-scale mass fluctuation. In this letter, we study the effect of tangled isotropic primordial magnetic fields on the collapse of a generic large scale spherical proto-object (representing a galaxy/galaxy group/galaxy cluster) by addressing the energetics of the collapse. This initial investigation leads to an intersting conclusion which is possibly quite robust. Our calculations indicate a strong constraint on the order of the magnetic field during re-combination - within the spherical approximation, we find that the collapse of objects is halted for fields ranging from $\gtrsim$3 nG for a protocluster to $\gtrsim$0.3 nG for a protogalaxy.

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1. Introduction

Magnetic fields are ubiquitous in the Universe. Fields at the level of micro-Gauss strength are present in collapsed structures like galaxies, clusters of galaxies etc [1]. Fields of nano-Gauss strength are inferred to be present in the intergalactic medium as well. The question of the origin of these fields is an important problem that is yet to find a convincing answer. The attempts at addressing this puzzle has led to two main scenarios viz the dynamo paradigm and the primordial field hypothesis. According to the dynamo mechanism, the present day magnetic fields in collapsed structures would have arisen out of an exponential amplification of a small seed field that might have been produced through any of the plasma processes in the early Universe [2]. This scenario has its own limitations though. Another equally interesting scenario is that the galactic/cluster fields have a primordial origin. According to the primordial field scenario, cosmic fields of the strength of nano-Gauss could through flux frozen evolution during a collapse amplify to micro-Gauss strength seen in these objects. This can be easily seen through the following order of magnitude argument. Assume a protoobject, with initial magnetic field at the recombination epoch ($z_{\rm i} \sim 1000$) $B_{\rm i}$, density $\rho_{\rm i}$(which is essentially the initial background density) collapses to form a structure (typically galaxy/cluster) through isotropic collapse. Assuming the collapse happens adiabatically, flux conservation in isotropic collapse implies $B \propto \rho^{2/3}$ and hence

$$\frac{B_{\rm col}}{B_i} = \frac{\rho_{\rm col}}{\bar{\rho}_{\rm i}} = \frac{\bar{\rho}_{\rm f}}{\bar{\rho}_{\rm i}}(1+\delta)^{2/3}$$

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Since, $B_{\rm i} = B_0 z_{\rm i}^2$ and $\bar{\rho} \propto z_{\rm i}^3$ because of cosmological expansion, we get

$$\frac{B_{\rm col}}{B_{\rm IGM}} = (\delta z_{\rm i})^{2/3}$$

since B_0 is simply $B_{\rm IGM}$, the intergalactic medium field at the current epoch. Putting in typical values $\delta \sim 10^3$ and $z_{\rm i} \sim 10^3$ we get $B_{\rm col} \sim 10^4 B_{\rm IGM}$. The primordial field scenario provides an appealing possibility of explaining the origin of galaxy/cluster fields and at this level is also consistent with the collapse logistics.

Another interesting thing to note is that the energy density of a primordial cosmic field of nano-Gauss strength is also at the same level(or slightly less) as the CMBR fluctuations since

$$\frac{\Delta T}{T} \sim \frac{B^2}{8\pi\rho_\gamma} \sim 10^{-6}$$

This is a curious coincidence. There have been several works addressed to the study of the detailed effects of primordial magnetic fields on CMBR [3;4]. From these studies, it can been generally concluded that B-fields induce scalar, vector as well as tensor perturbations on the background. The scalar and tensor modes are dominant on large scales whereas vector modes have a predominant effect on small scales.

The effects of a primordial field on the matter power spectrum have also been studied on large scales using the linear perturbation theory [5;6;7]. It has been seen that B-fields existing at recombination epoch can provide the initial conditions for the generation of density perturbations [6;7]. Tangled B-fields with a scale-invariant power spectrum predominantly induce matter power on small scales. This behaviour persists as long as the fluctuations are in the linear regime. However, to address the question of the non-linear behaviour which would result in the collapse to a viralized state, it is important to consider the energetics of the problem.

In the standard cosmological scenario, the formation of the collapsed objects is an intricate problem since the evolution is non-linear. It is mainly understood through N-body simulations combined with scaling arguments. In this context, the spherical collapse model is a useful analytic model to trace the evolution of an initial isolated spherical density perturbation well into the non-linear regime. In this exercise, we study the cosmological evolution of a spherical perturbation in the scenario when primordial magnetic fields are present.

2. Formulation & Results

To study the spherical collapse of matter in the presence of primordial magnetic fields, we first note that magnetic fields only affect ionized matter, and hence there is no direct effect on dark matter. However since the magnetic fields are frozen into ionized baryonic matter and baryonic matter is strongly coupled to dark matter gravitationally we can treat the magnetic fields to be frozen into a combined (dark+baryonic) matter fluid. The resulting treatment then closely parallels the standard scenario [8]. Consider the cosmological evolution of a mass shell with Lagrangian coordinate $r = r(t)$. The equation of motion of the mass shell is given by :

$$\frac{d^2r}{dt^2} = -\frac{G\mathrm{M}(r)}{r} + \frac{(\nabla \times B \times B)_r}{4\pi\rho} \tag{1}$$

$\mathrm{M}(r)$ is the net mass contained within the radius r. The equation of motion of a mass shell gets modified by the addition of the Lorentz force due to the magnetic field. The subscript r on the right hand side for the Lorentz force denotes the radial component. The Lorentz force on the RHS in general depends on the detailed field configuration. If the magnetic field is completely tangled on the scales we are interested in, the Lorentz force is gradient of the magnetic pressure only. In this case we have,

$$\frac{d^2r}{dt^2} = -\frac{G\mathrm{M}(r)}{r^2} - \frac{1}{8\pi\rho}\frac{dB^2}{dr} \tag{2}$$

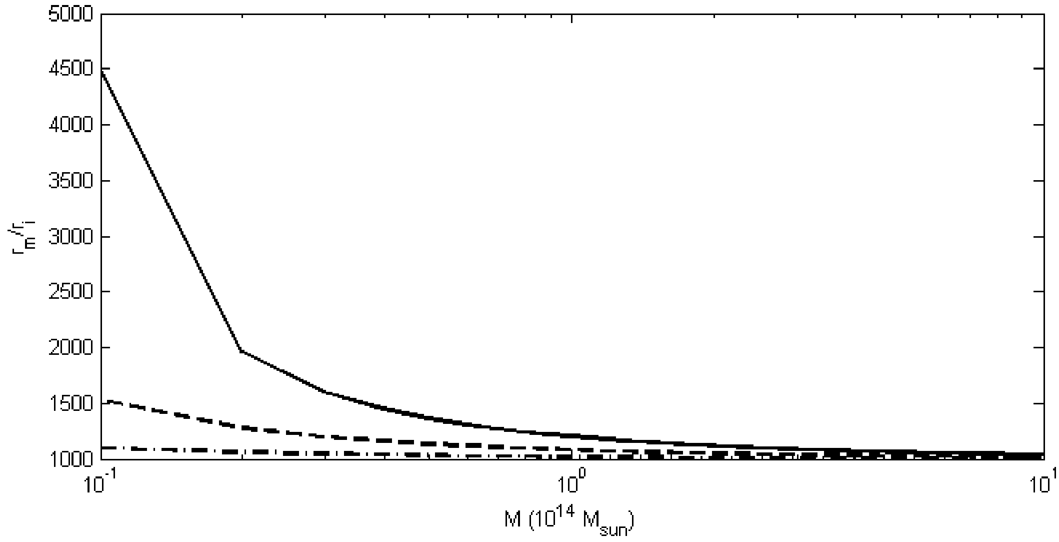

Figure 1: Turnaround ratio $r_{\mathrm{m}}/r_{\mathrm{i}}$ versus mass (in units of $10^{14}\,\mathrm{M}_{\odot}$ for different magnetic field strengths. The dark, dashed and dot dashed curves correspond to field strengths of $0.9B_{\mathrm{max}}$, $0.5B_{\mathrm{max}}$ and $0.1B_{\mathrm{max}}$ respectively

Since the conductivity of the fluid is high on cosmological scales, the magnetic field evolves in such a way that the flux is conserved. This implies that

$$Br^2 = constant = B_i r_i^2 \tag{3}$$

From the above relation it can be seen that the Lorentz force term on the RHS has the same r dependance as that of the gravitational term since i.e $\propto r^{-1}$. The equation of motion can hence be rewritten as:

$$\frac{d^2r}{dt^2} = -\frac{G\mathrm{M}^{\mathrm{B}}(r)}{r^2} \tag{4}$$

where the equivalent mass M^B is :

$$\mathrm{M}^{\mathrm{B}} = \mathrm{M} - \frac{2}{3}\frac{B_{\mathrm{i}}^2 r_i^4}{G\mathrm{M}} \tag{5}$$

Thus as far as equation of motion is concerned, the evolution of the radius of the mass shell within which there is matter of mass M and initial magnetic field B_i occurs exactly equivalent to that of a shell inside which there is an equivalent mass M^B Assuming that the mass shells do not cross, the solution to the above which is essentially the energy equation can be written as:

$$E = \frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{G\mathrm{M}^{\mathrm{B}}}{r}, \tag{6}$$

where the constant E is the conserved total energy of the shell. At an initial time t_{i}, the shell is considered to be expanding alongwith the Universe. Here, we assume that this initial time is immediately after the recombination epoch. In this case, the velocity of the shell is simply the Hubble velocity i.e $(dr/dt)_{\mathrm{i}} = H_{\mathrm{i}} r_{\mathrm{i}}$ and the mass of the shell in terms of the background density ρ_{bi} and the initial perturbation δ_{i} can be written as :

$$\mathrm{M} = \frac{4}{3}\pi r^3 \rho_{\mathrm{b}}(1+\delta_{\mathrm{i}}) \tag{7}$$

Using the Friedmann equation for a flat background Universe $H^2 = 8\pi G\rho_b/3$ the energy at t_{i} can be expressed as:

$$E \equiv E_{\mathrm{i}} = -\frac{G\mathrm{M}\delta_{\mathrm{i}}}{r_{\mathrm{i}}(1+\delta_{\mathrm{i}})} + \frac{B_{\mathrm{i}}^2}{2\pi\rho_{\mathrm{bi}}(1+\delta_{\mathrm{i}})} \tag{8}$$

The energy E can be positive, negative or zero depending on the relative strength of the two terms on the RHS. If E is negative, the region within the mass shell evolves like a closed Universe. As a result, the radius of the shell initially increases, reaches a maximum (denoted by r_{m}) and then decreases. In such a case a collapsed object can form. However if E is zero or positive, then the radius of the overdense region keeps increasing forever and collapse cannot occur. Hence the fact that the total energy has to be negative leads to a strong constraint on the possible values of the magnetic field strength. From the expression for total energy (Eq. 8) the upper limit can be evaluated to be:

$$\begin{aligned} B_{\mathrm{max}} \simeq & 0.3\mathrm{mG} \times \left(\frac{\mathrm{M}}{10^{11}\mathrm{M}_{\odot}}\right)^{1/3}\left(\frac{\Omega_{\mathrm{m}}}{0.3}\right)^{2/3} \\ & \times \left(\frac{h}{0.7}\right)^{4/3}\left(\frac{1+z_{\mathrm{i}}}{1000}\right)^{2}\left(\frac{\delta_{\mathrm{i}}}{10^{-3}}\right)^{1/2} \end{aligned} \tag{9}$$

The above limit for the initial magnetic field translates to a comoving current epoch field strength of

$$B_0 = \frac{B_{\mathrm{i}}}{(1+z_{\mathrm{i}})^2} \simeq 0.3\mathrm{nG}$$

for fiducial values given above. Note that the above limit can be considered as the root mean square field (i.e $B_{rms} = B_i$ on the scale r_i).

3. Discussion

This limit is stronger than the limits obtained from CMBR for a galaxy sized fluctuation. Thus for fields less than the above critical value, the spherical region enclosing a given mass, initially expands, reaches a maximum radius and then breaks away from expansion to collapse to a final virialized state. It is instructive to track the subsequent evolution of the radius of the shell enclosing the mass M to evaluate the effect of the sub-critical fields on the turnaround ratio i.e the fraction by which the radius of the region expands from its initial size to maximum size. The evolution of the shell upto the maximum radius occurs with the mass within the shell being conserved. When the shell attains maximum radius r_{m}, the velocity is zero, hence we have,

$$E \equiv E_{\mathrm{m}} = -\frac{G\mathrm{M}}{r_{\mathrm{m}}} \tag{10}$$

The mass conservation relation also gives:

$$\mathrm{M} = \frac{4}{3}\pi r_{\mathrm{i}}^3 \rho_{\mathrm{bi}}(1+\delta_{\mathrm{i}}) = \frac{4}{3}\pi r_{\mathrm{m}}^3 \rho_{\mathrm{bm}}(1+\delta_{\mathrm{m}}) \tag{11}$$

where δ_{m} and ρ_{bm} are the density contrast and the background matter density at maximum radius. Using eqns (8), (9), (10), we thus find the following relation:

$$\frac{r_{\mathrm{m}}}{r_{\mathrm{i}}} = 1 - \frac{G\mathrm{M}}{E_{\mathrm{i}} r_{\mathrm{i}}} \tag{12}$$

In figure (1), we have plotted the turnaround radius ratio $r_{\mathrm{m}}/r_{\mathrm{i}}$ as a function of mass M for different values of the magnetic field strengths below B_{max}. It can be seen that the maximum effect of the magnetic field occurs for lower mass fluctuations and the effect is to increase the turnaround ratio. This is because for lower mass fluctuations for a given sub-critical field, the difference between inward acting gravitational force and the outward acting magnetic force is relatively less. As a result, the shell enclosing a particular mass moves much further out before it can turnaround to start collapse.

Thus in this paper, we have investigated the possibility of non- linear collapse of a large spherical protoobject in the presence of a fully tangled magnetic field within the region through the spherical collapse model. In particular, we investigated the energy logistics and found that a fully tangled isotropic magnetic field impedes the collapse of a cosmic proto-object. We also derived a maximum limit for the magnetic field as function of mass and found that for galaxy cluster sized perturbations, comoving field strengths greater than $\sim$ 3nG will not let the collapse to happen and for galaxy-sized perturbations the upper limit decreases by an order of magnitude to 0.3nG.

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