MIXING AND DECOHERENCE TO NEAREST SEPARABLE STATES

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Introduction

- Studying environment(**E**) induced decoherence on a bipartite composite system where one of the subsystems (**S**) is in pure state and the other (**A**) is in a mixed state.
- This can be considered as a model for measurement process where **S** is the system and **A** is the apparatus.
- The apparatus has to be considered classical in the sense that it is an open system with a large number of degrees of freedom interacting weakly with an enormous number of environmental variables[see ref 2].
- Proposing a principle generalising a similar one [see ref 1] with both the subsystems in pure state, which may give the *correct* measurement statistics.

Pre-measurement process

Orthogonal states $|s_1\rangle$, $|s_2\rangle$ of a two-state system (S), being eigenstates of an observable, say, $\hat{S} \equiv s_1|s_1\rangle\langle s_1| + s_2|s_2\rangle\langle s_2|$, get correlated with two mixed apparatus states, say, $\rho_a^{(A)}$ and $\rho_b^{(A)}$ respectively, each corresponding to a definite value of the relevant **pointer variable** [see ref 3,4], such that

$$\rho_a^{(A)} = \sum_{i=1}^{N_1} p_i |a_i\rangle\langle a_i|,$$

$$\rho_b^{(A)} = \sum_{i=1}^{N_2} q_i |b_i\rangle\langle b_i|,$$

Pre-measurement process(contd.)

The density matrix of the *composite system* becomes,

$$\rho = |c_1|^2 |s_1\rangle\langle s_1| \otimes \rho_a^{(A)} + |c_2|^2 |s_2\rangle\langle s_2| \otimes \rho_b^{(A)}$$
$$+ c_1 c_2^* |s_1\rangle\langle s_2| \otimes |\phi_a^{(A)}\rangle\langle\phi_b^{(A)}| + c_1^* c_2 |s_2\rangle\langle s_1| \otimes |\phi_b^{(A)}\rangle\langle\phi_a^{(A)}|.$$

where,

$$|\phi_a^{(A)}\rangle = \sum_{i=1}^{N_1} p_i |a_i\rangle,$$
$$|\phi_b^{(A)}\rangle = \sum_{i=1}^{N_2} q_i |b_i\rangle.$$

Offdiagonal terms are fixed to get a legitimate ρ after purification[1] from a pure state of a higher dimensional space.

Nearest Separable State

In terms of relative entropy,

$$S(\rho_1|\rho_2) = \mathrm{Tr}\rho_1(\mathrm{ln}\rho_1 - \mathrm{ln}\rho_2,)$$

the state

$$\rho^* = \mathrm{diag}(\rho)$$

can be shown as the nearest among all the separable states. To show that consider the function $f(x) = S(\rho|(1-x)\rho^* + x\sigma)$ for an arbitrary separable state σ. After suitable manipulation it comes as,

$$\frac{d}{dx}f(x)\Big|_{x=0} > 0.$$

Nearest Separable State (contd.)

Thus $S(\rho|\rho^*)$ takes locally the least value for ρ^* and using the convexity property of relative entropy it can be shown that it is the global least value among all the disentangled states for this specific choice of ρ.

Now if

$$\rho_S^* = \mathrm{Tr}_A(\rho)$$
$$\rho_A^* = \mathrm{Tr}_S(\rho).$$

$S(\rho|\rho^*)$ can be thought of as the quantum part (i.e. due to quantum entanglement) of the the total correlation $S(\rho|\rho_S^* \otimes \rho_A^*)$ between ρ and product state of the reduced density matrices of its subsystems[ref.5,6].

An estimate of quantum correlation

Let us consider a solvable case where $p_i = 1/N_1$ and $q_j = 1/N_2$ for all values of the indices, i.e.,for $\rho_a^{(A)}$ and $\rho_b^{(A)}$ to be maximally mixed states in the respective subspaces. It comes that,

$$S(\rho|\rho^*) = \frac{|c_1|^2}{N_1}\ln[1+\frac{|c_2|^2 N_1}{|c_1|^2 N_2}] + \frac{|c_2|^2}{N_2}\ln[1+\frac{|c_1|^2 N_2}{|c_2|^2 N_1}],$$

where it is apparent that this quantum part of the correlation diminishes with the larger values of degeneracy of pointer states, i.e. of N_1 and N_2.

On the other hand the relative entropy between ρ^* and $\rho_S^* \otimes \rho_A^*$ gives the classical part of the total correlation.

Decoherence process: Hamiltonian

The total Hamiltonian taking account of the environment, is taken as,

$$H = H_S \otimes I_A \otimes I_E + I_S \otimes H_A \otimes I_E + I_S \otimes I_A \otimes H_E + \lambda I_S \otimes V_{A-E}.$$

1. The premeasurement interaction between system and apparatus is ignored here due to its assumed shorter timescale to bring the **S-A** composite system into the state ρ.
2. As the apparatus is a macroscopic system with large degrees of freedom and the environment with innumerable degrees of freedom must interact chaotically, the interacting Hamiltonian(V_{A-E}) between the apparatus(A) and the environment(E) has to be a real symmetric random matrix.

Decoherence: Time Evolution

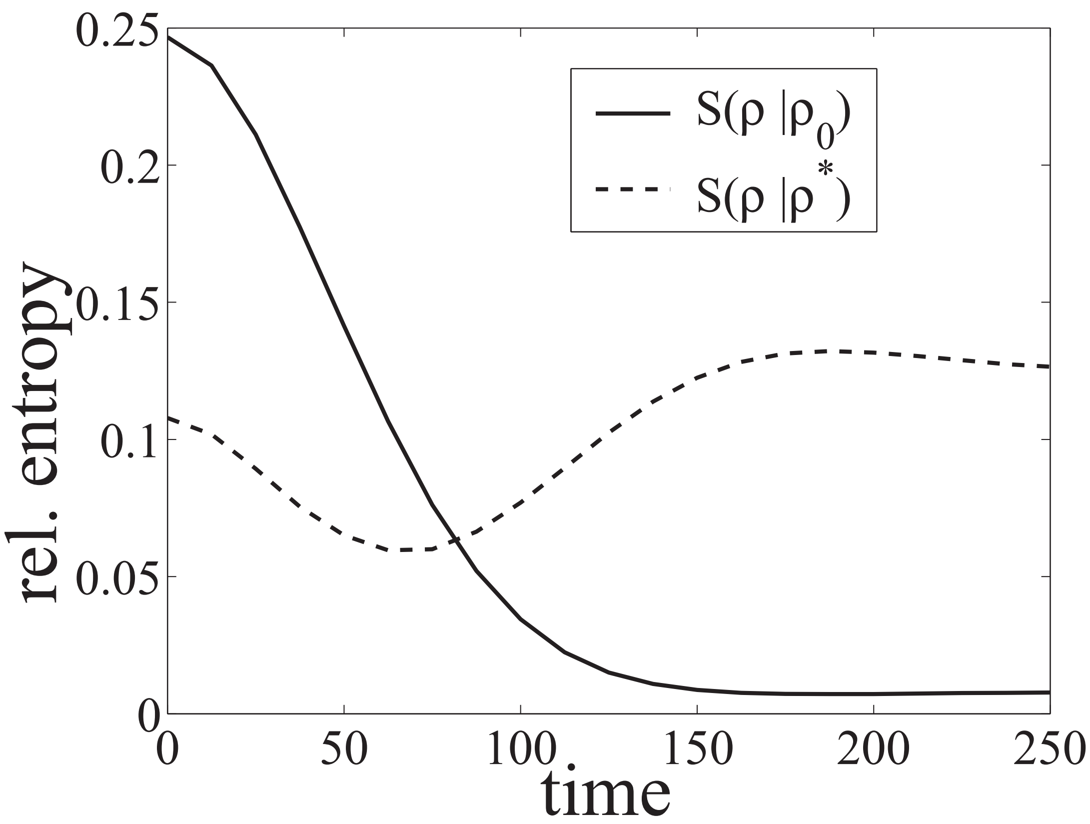
Numerical investigation about time evolution reveals two facts:

- The offdiagonal elements of reduced density matrix of the composite system S-A decays with time, i.e. the composite system decohere with time due to enviroment.
- Along with that process the there taking place a mixing among the apparatus states within each group. Though that mixing can be avoided using non-demolition type of coupling with the environment.

Thus generally the final state does not become the decohered state ρ^* but an equimixed dehohered state ρ_0 which can be found by putting $p_i = 1/N_1, q_j = 1/N_2$ for all i, j's in the expression of ρ^*.

Numerical Evidences

The figure shows the time evolution of distances of reduced state of S-A from ρ^* and ρ_0.

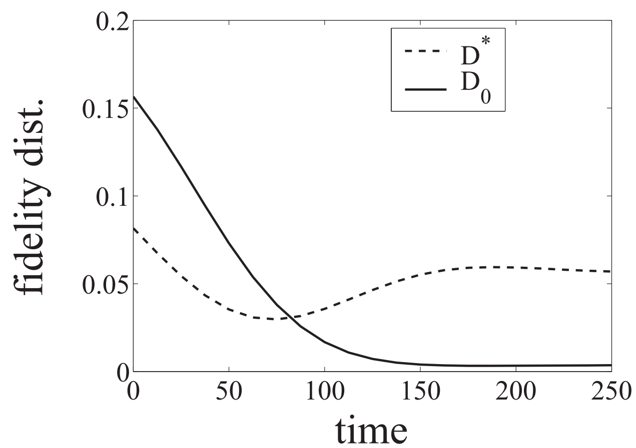


Numerical Evidences (contd.)

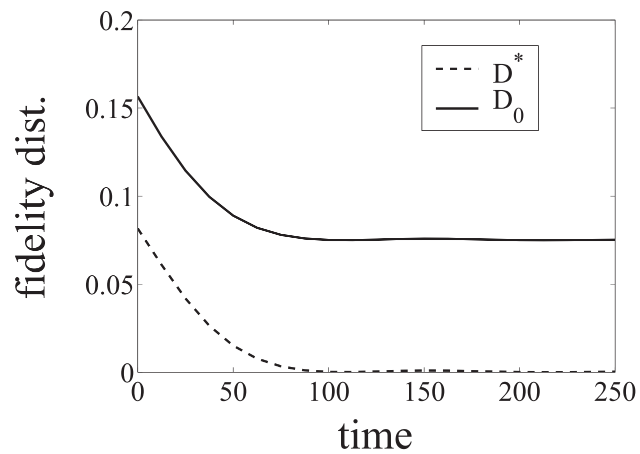
Bures metric[ref.1] or fidelity distance provides one with another distance function and is defined by,

$$D_B(\rho|\rho^*) = 2 - 2\sqrt{F(\rho|\rho^*)}$$

where $F(\rho|\rho^*) = [\mathrm{Tr}(\sqrt{(\rho^*)}\rho\sqrt{(\rho^*)})^{1/2}]^2$.



(a) Random matrix coupling



(b) Nondemolition coupling

Conclusions

- The pure-mixed premeasurement state, due to deoherence caused by the environment, tends to reach the nearest separable state.
- But at the same time pointer states tend to equimix within the groups corresponding to distinct pointer values,but across the groups there happens no mixing in the relevant time scale.
- When the mixing becomes totally suppressed the decoherence process ultimately ends up at the nearest separable states.
- These principles of decoherence and mixing conforms with the correct measurement statistics.

The detailed calculations and more exhaustive numerical evidences will be found in:A. Lahiri, G. Ghosh and S. Nag,*"Int. Jour. of Quant. Inf."*,**7**,829–846(2009).

References

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