Development of Secular Instability in Different Disc Models of Black Hole Accretion

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Introduction

- There are observational evidences in support of black hole accretion discs in AGN scheme.
- Though such astrophysical accretion process is expected to be highly violent and turbulent one, at large length scale it is assumed to be stationary.
- But the stability of such a stationary flow is a requirement for formation of such a disc.
- In the present work the stability of the accretion disc is investigated in quasi-viscous regime under pseudo-potential scheme.

Black Hole Accretion

- Accreting black holes as the central engines are widely accepted model of Active Galactic Nuclei (AGN).
- The continuous stream of infalling matter with angular momentum forms a disc around a compact object, a black in this case.
- There are different proposed models of disc structures available in the literature.[e.g. ref 1]
- The matter falling into the black hole must start from large distance with very little radial velocity and must arrive at the event horizon with supersonic speed – the accretion flow must be transonic.

General Assumptions

In an analytical or semi-analytical treatment few assumptions are made, such as:

Hydrodynamic Though the matter becomes ionised at that extreme condition where MHD treatment is more realistic, due to analytical convenience Hydrodynamic calculations are used to find rough estimates of flow characteristics.

Non-self-gravitating As the density of the accreted matter is very very small, the gravitational effect of the matter on itself is being neglected.

Small Viscosity We are investigating the low angular momentum regime of the accreted matter, where very little viscous effect may be expected to be present.

Stationary Flow

It is generally accepted that the accretion flow eventually reach stationary configuration.

But for that to happen a necessary condition is: the stationary configuration must be stable under a time dependent perturbation.

A number of authors within last decade or so analytically proved that this condition is obeyed at least in inviscid flow.[e.g. ref 1. and references therein]

But recently a couple of works demonstrated that this condition is violated with the emergence of slightest amount of viscosity introduced in quasi-viscous form. Calculations were done there in pseudo-potential framework.[ref 2]

Pseudo-Potential

Instead of using general relativistic framework, which is not easy to do in this case, we used pseudo-Newtonian framework. where,

1. Newtonian force law or potential formulation is taken to be valid.
2. But the potential ϕ is tinkered in such a way that it may mimic the actual motion in curved space-time.
3. Here we used pseudo-potential ϕ for non-rotating black hole.

Viscous Flow

The governing flow equations in a disc geometry are continuity equation,

$$\frac{\partial}{\partial t}(\Sigma) + \frac{1}{r}\frac{\partial}{\partial r}(\Sigma v r) = 0,$$

where Σ denotes mass per unit area of the disc, v denotes radial inward velocity along with Euler equations in radial direction and in azimuthal direction.

The azimuthal equation may be expressed as

$$\rho H \frac{\partial}{\partial t}(r^2\Omega) + \rho v H \frac{\partial}{\partial r}(r^2\Omega) = \frac{1}{2\pi r}\left(\frac{\partial G}{\partial r}\right),$$

where, $G = 2\pi\nu\Sigma r^3 \dfrac{\partial \Omega}{\partial r}$ is the viscous torque with ν as kinematic viscosity.

Quasi-viscous Approximation

The radial equation takes the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \Phi^{'}(r) - \frac{\lambda_{eff}^2}{r^3} = 0,$$

where,

$$\lambda_{eff} = r^2\Omega = \lambda_0 + \alpha r^2 \tilde{\Omega}(c_s, v, r),$$

for $\alpha = \nu c_s H$. H varies from one disc model to another, c_s being the sound speed.

Now, as $\alpha << 1$, it can be approximated as

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \Phi^{'}(r) - \frac{\lambda_0^2}{r^3} - \frac{2\alpha\lambda_0 F(c_s, v, r)}{r^3} = 0$$

Stationary Equations

Stationary condition means $\frac{\partial}{\partial t} = 0$
Thus the radial equation becomes,

$$v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \Phi^{'}(r) - \frac{\lambda_0^2}{r^3} - \frac{2\alpha\lambda_0 F}{r^3} = 0$$

and from the contnuity equation,

$$f \equiv \rho v r H, \text{ constant over } r.$$

Thus the effect of viscosity turns out to be a perturbation, linear in viscosity parameter α, on the inviscid case i.e. when $\alpha = 0$.

Time Dependent Perturbation

Consider small time dependent perturbation over the stationary solution of the previous equation as,

$$v(r,t) = v_0(r) + \tilde{v}(r,t)$$
$$\rho(r,t) = \rho_0(r) + \tilde{\rho}(r,t)$$
$$f(r,t) = f_0(r) + \tilde{f}(r,t)$$
$$F(r,t) = F_0(r) + \tilde{F}(r,t)$$

Put these variables into the general time dependent equation to find the differential equation regarding the time evolution of the perturbing terms.

Time Evolution of Perturbations

Retaining the the linear perturbing terms only one gets:

$$\partial\partial t\left[\frac{v_0}{f_0}\frac{\partial\tilde{f}}{\partial t}\right]+\frac{\partial}{\partial t}\left[\frac{v_0^2}{f_0}\frac{\partial\tilde{f}}{\partial r}\right]+\frac{\partial}{\partial r}\left[\frac{v_0^2}{f_0}\frac{\partial\tilde{f}}{\partial t}\right]$$
$$+\frac{\partial}{\partial r}\left[\frac{v_0}{f_0}\left(v_0^2-\frac{c_{s0}^2}{1+\varepsilon}\right)\frac{\partial\tilde{f}}{\partial r}\right]-\frac{2\alpha\lambda_0^2}{r^3}\frac{\partial\tilde{F}}{\partial t}=0$$

where ϵ is a disc model dependent parameter and

$$\frac{\partial\tilde{F}}{\partial t}=\frac{c_{s0}}{\rho_0 v_0 r^2}\frac{(1+\gamma+4\varepsilon)}{(1+\varepsilon)}\frac{\partial\tilde{f}}{\partial r}+2\int\frac{\partial}{\partial r}\left(\frac{c_{s0}}{\rho_0 v_0^2 r^2}\right)\frac{\partial\tilde{f}}{\partial t}dr$$

where the polytropic equation of state is given by $P=K\rho^{\gamma}$

Different Disc Models

Three disc models are generally available in the literature [ref. 3]:

Vertical Equilibrium

$$H = c_s \sqrt{\frac{r}{\gamma \phi'}}, \text{ hence } \epsilon = (\gamma - 1)/2.$$

Constant Height

$$H = \text{constant}, \text{ hence } \epsilon = 0$$

Conical Flow

$$H = \Theta r, \text{ hence } \epsilon = 0$$

Stationary Wave Solution

For stationary wave solution

$$\tilde{f}(r,t) = g_\omega(r)e^{-i\omega t}$$

Upon integrating we get finally,

$$A\omega^2 + B\omega + C = 0$$

$$A = \int v_0 g_\omega^2 dr \quad B = -4i\alpha\lambda_0^2 \int \frac{g_\omega f_0}{r^3}\left[\int g_\omega \frac{d}{dr}\left(\frac{c_{s0}}{\rho_0 v_0^2 r^2}\right) dr\right]$$

$$C = \int v_0 \left(v_0^2 - \frac{c_{s0}^2}{1+\varepsilon}\right)\left(\frac{dg_\omega}{dr}\right)^2 dr$$

$$+2\alpha\lambda_0^2 \int \frac{c_{s0} f_0}{\rho_0 v_0 r^5}\left(\frac{1+\gamma+4\varepsilon}{1+\varepsilon}\right) g_\omega \frac{dg_\omega}{dr} dr$$

Asymptotic Behaviour

It turns out,

$$Re(-i\omega) =\sim \alpha\xi(r)$$

where

For Constant Height disc

$$\xi(r) \sim r^{-2}$$

For Conical Flow disc,

$$\xi(r) \sim r^{1}$$

For Vertical Equilibrium disc,

$$\xi(r) \sim r^{5/2}$$

Conclusions

- Other than Constant Height model, for two other models the perturbation grows asymptotically making the disc unstable.
- But the source term of instability is proportional to viscosity parameter α.
- Hence for a sufficiently low value of α the perturbation grows in a very large time scale.
- This may make the disc effectively stable in Astrophysical perspective

References

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