A Study on the Chaotic Motion of a Test Particle near a Spinning Black Hole with Dipolar Halo

Sankhasubhra Nag

sankhasubhra_nag@yahoo.co.in

Department of Physics

Sarojini Naidu College for Women

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Spinning Black Hole with Halo

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For a test particle in addition to the strong influence of the black hole, the halo also offers some weak perturbation

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A spinning black hole means a black hole with some intrinsic angular momentum

The space-time structure bears the signature of the spin of the black-hole, which is called the Kerr space-time

A More Simplified Approach

General Relativistic calculations cannot be done without some gross approximations.
Hence an equally valid but analytically easier to handle, method is the use of an suitably chosen effective potential in Newtonian scheme to mimic the actual motion in a smaller region of space.

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For spinning black such a prescribed pseudo-potential is

$$\phi(r) = \frac{1}{r_1(\beta - 1)} \left[1 - \frac{r^{\beta-1}}{(r - r_1)^{\beta-1}} \right]$$

where β and r_1 are some parameters solely dependent on spin paramter a

A More Simplified Approach

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In the first approximation i.e. dipole approximation the effect of Halo is captured by a perturbing potential αz.

The particle is assumed to have a initial angular momentum L.

In the cylindrical polar co-ordinate the equation motion is setnear equatorial plane using Newton's Law for four dynamical variables r, p_r, z and p_z.

Hamiltonian Systems

The systems , governed by the Hamilton's Equations i.e.

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \;\; \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

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are called Hamiltonian systems. The chaos in Hamiltonian system is called Hamiltonian chaos.

If one starts from a distribution of points with distribution function $\rho(p_i, q_i)$, during time evolution

$$\frac{d\rho}{dt} = 0;$$

Total phase space volume of the distribution of points remains constant in time.

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Total phase space volume of the distribution of points remains constant in time.

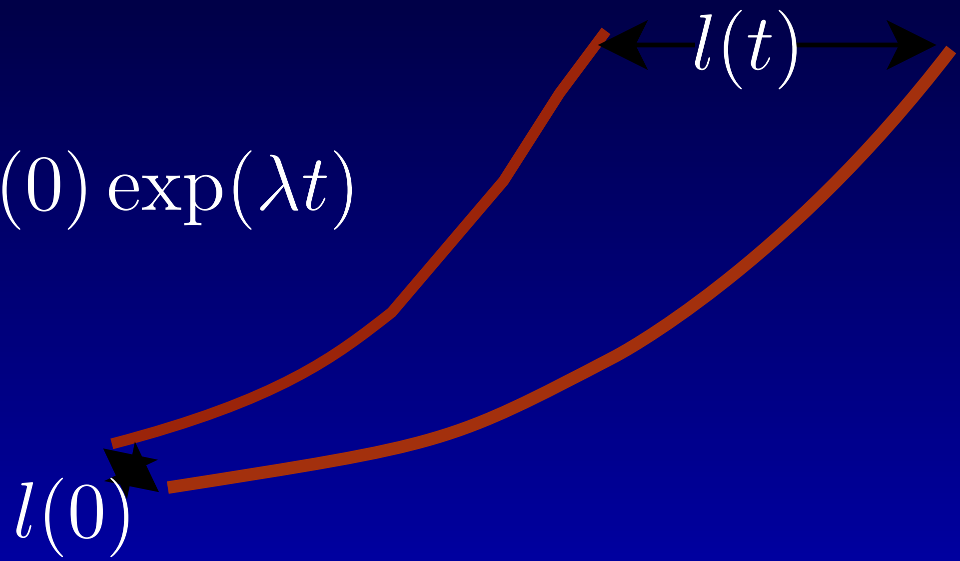
It excludes the existence of attractor or repelling nodes in these systems.

Chaos

Some bound motion extremely sensitive to initial conditions is called chaotic.

i.e.

$$l(t) = l(0)\exp(\lambda t)$$



λ is called the *Lyapunov exponent*, which is a measure of chaos

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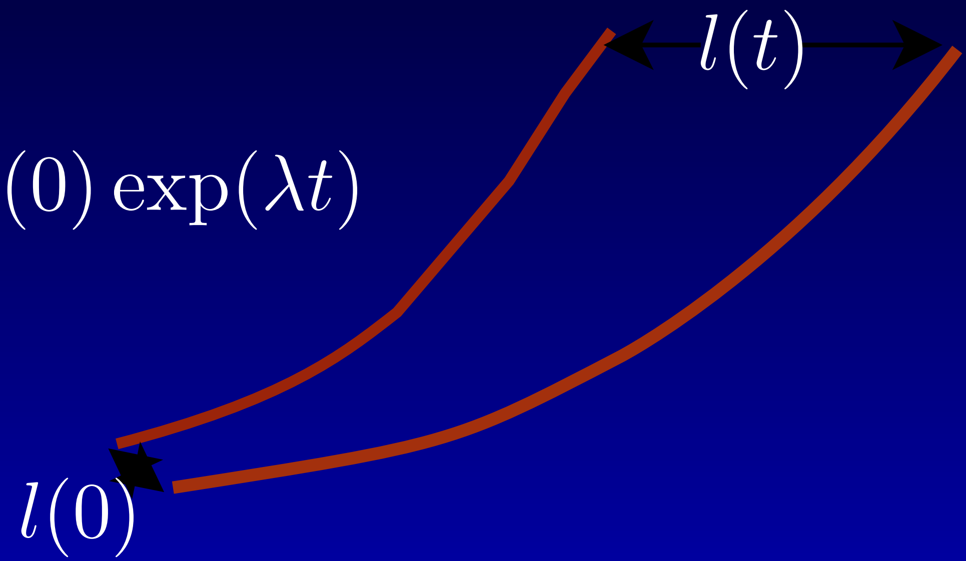


Though chaos is a general concept valid for any type of dynamical system, for the present context we restrict ourselves within the motion of particles under a force field i.e. within the **Hamiltonian systems**.

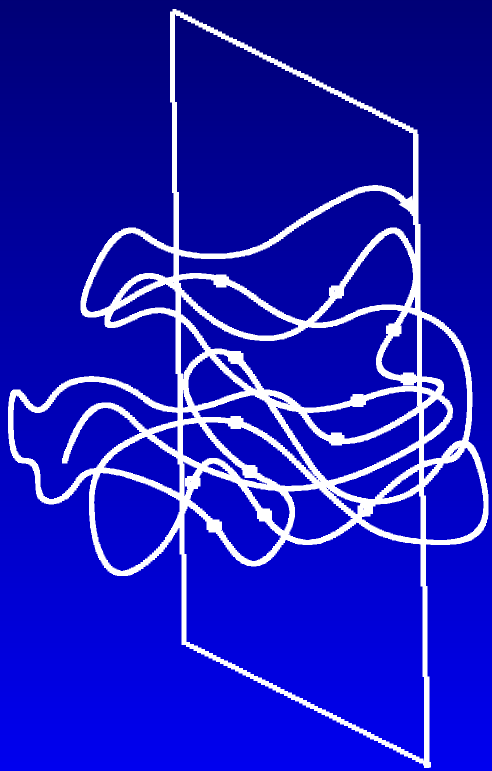
Chaotic trajectories

For time independent Hamiltonian the trajectories are restricted within the constant energy subspace but –

The trajectories wander around the accessible

phase space with various twists and turns to

diverge away from the vicinity of the others



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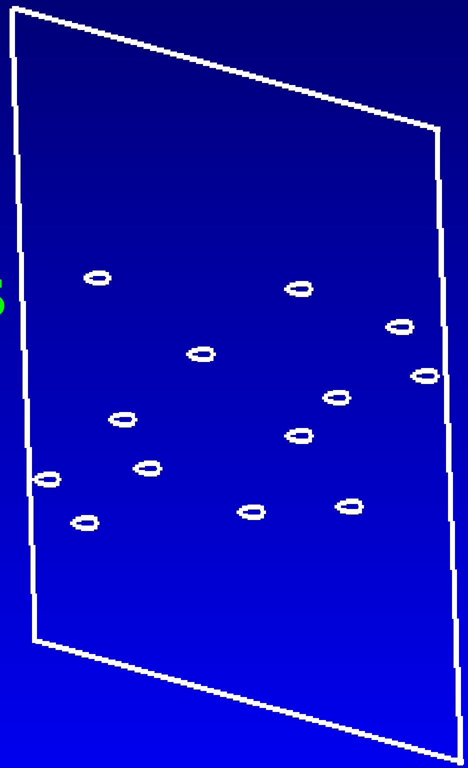
the crosssection of the trajectories in lower dimensional plane thus generates scattered points without any pattern characteristic to chaos



Integrable Hamiltonian Systems

Hamiltonian systems are called integrable only when there exist N independent constants of motion

$$f_m(q,p) = c_m \quad m = 1, ..., N$$

and also

$$[f_m, f_n] = \sum_i^N \frac{\partial f_m}{\partial p_i}\frac{\partial f_n}{\partial q_i} - \sum_i^N \frac{\partial f_m}{\partial q_i}\frac{\partial f_n}{\partial p_i} = 0$$

for all $m = 1, ..., N$ & $n = 1, ..., N$

These two conditions ensure that motion of the system in phase space will be on some toroid like object

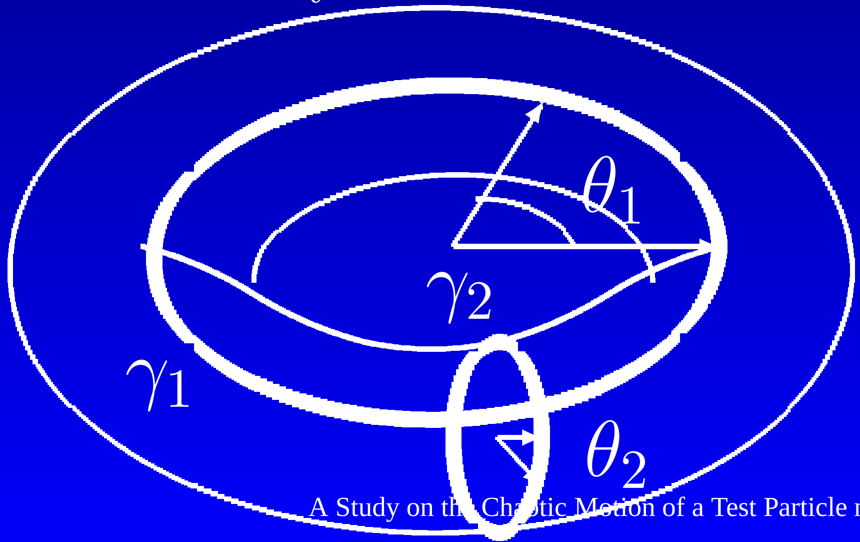
Motion in Phase Space

For the canonical transformation,

$$\begin{pmatrix} q \\ p \end{pmatrix} \longrightarrow \begin{pmatrix} \theta \\ J \end{pmatrix}$$

where

$$2\pi J_k = \sum_i \oint_{\gamma_k} p_i dq_i = J_k(f = c)$$

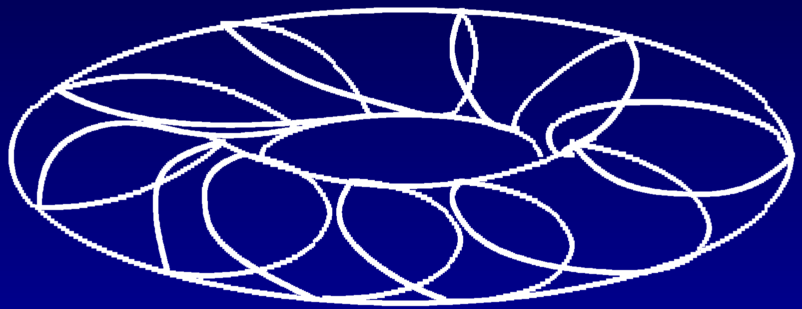


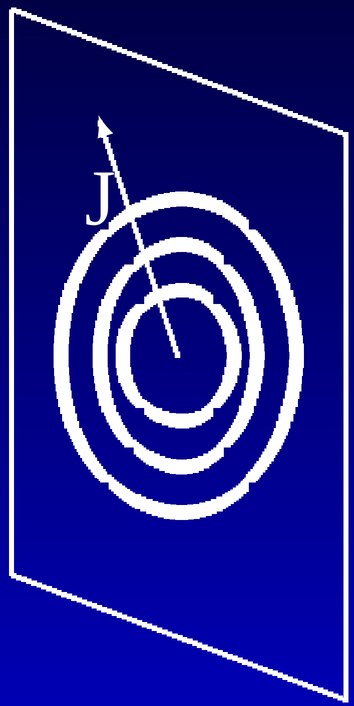
Motion in Phase Space

Such that

$$H \longrightarrow H(J)$$

and





The motion on the torus will be either periodic, or it will be quasiperiodic.

The Chaotic Motion in our case!

The Poincare section and Liapunov exponent of the test particle is found as

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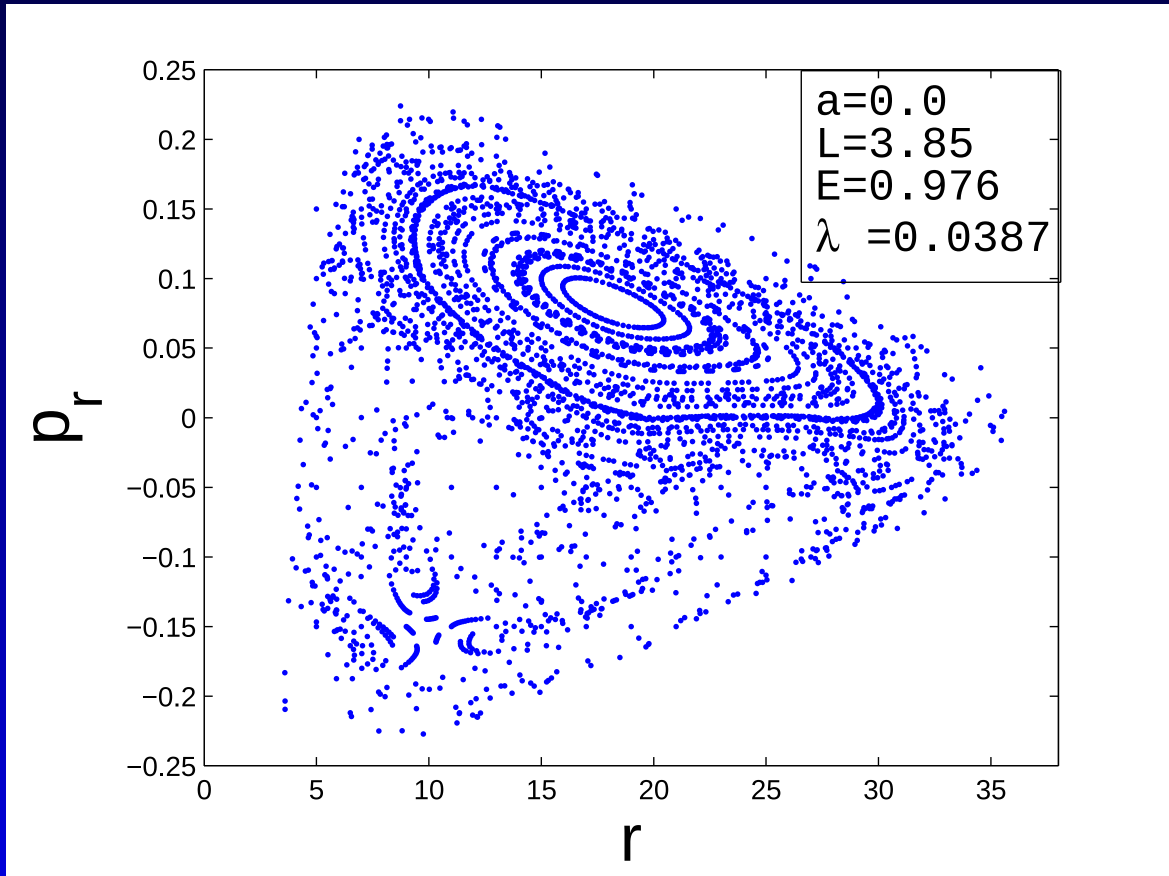

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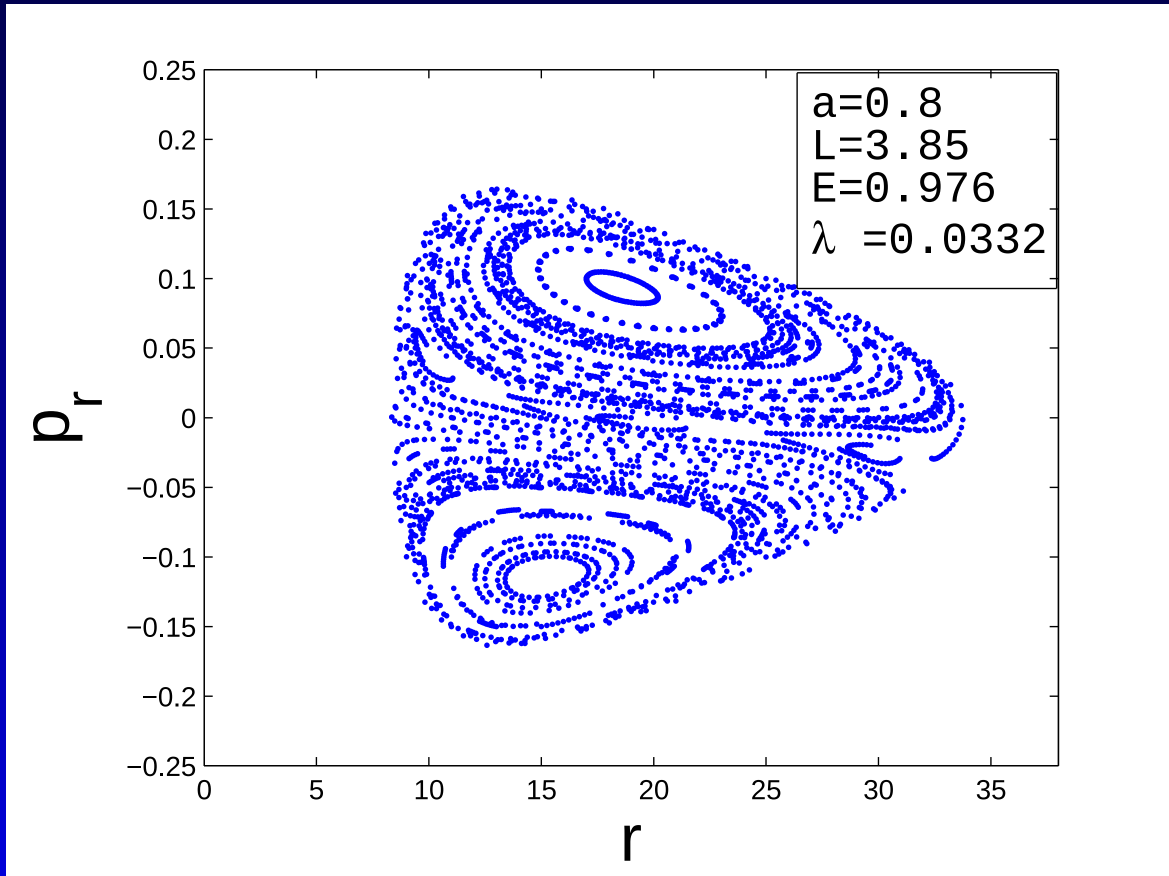

The End

Collaborators:

1. Dr. Siddhartha Sinha, Sarojini Naidu College for Women, Dumdum
2. Dr. Tapas Kumar Das, Harischandra Research Institute, Allahabad

Thank You!