Studies on Multi-transonic behaviour of the Black Hole Accretion using Techniques of Dynamical Systems

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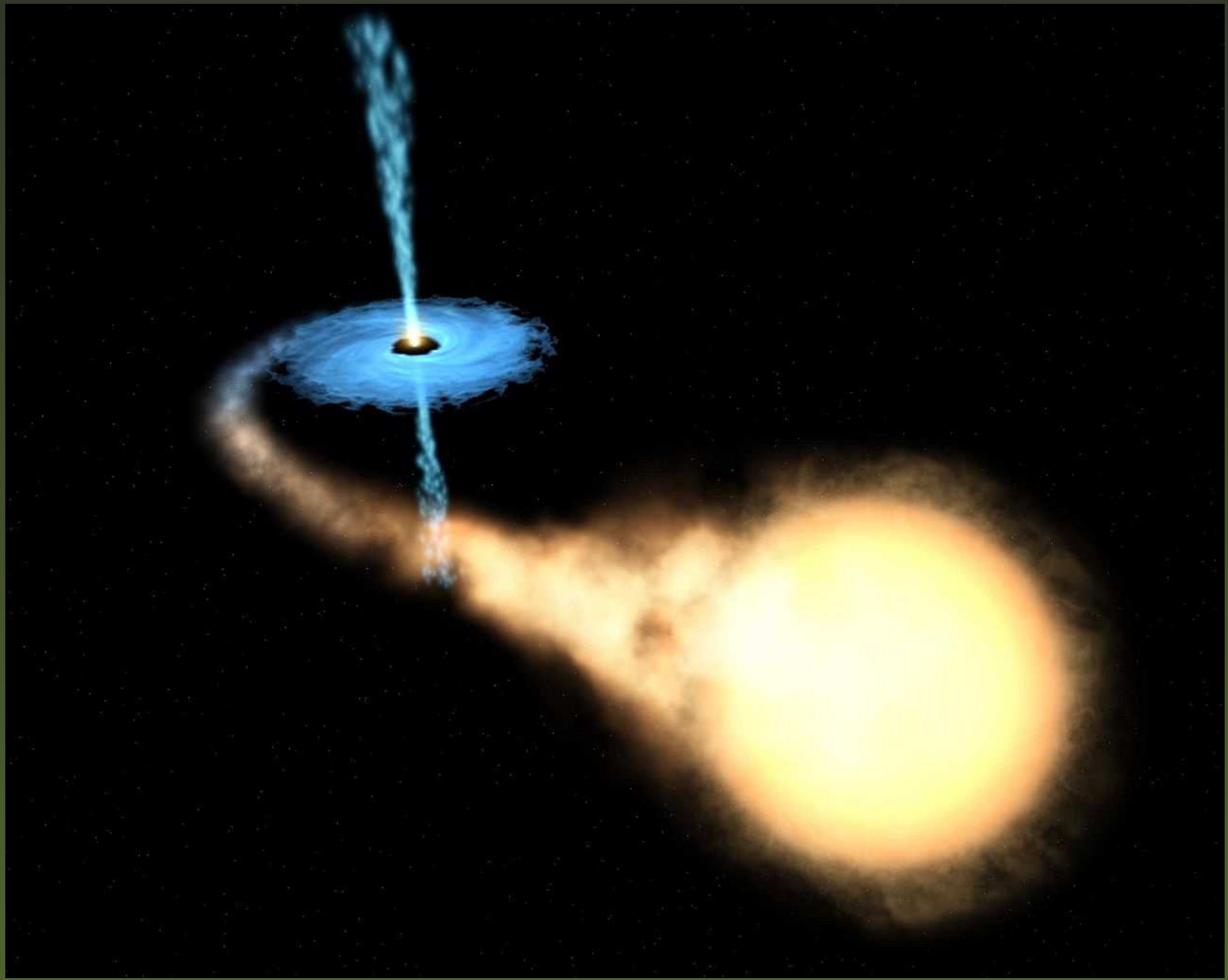
Accretion in Astrophysics



Accretion in Astrophysics

In our specific case:

- Inviscid Hydrodynamic flow.
- Axisymmetric Low Angular momentum.
- Barotropic flow.
- Non-self Gravitating Fluid.

The governing equations are

1. Euler Equation.
2. Equation of Continuity.
3. Equation of State.

Space-Time Property

Though near Black Holes space-time is curved,i.e. it needs General Relativity; to begin with we assume a flat space-time where Newtonian scheme is approximately valid with suitably tinkered potentials $\phi(r)$ which are called pseudo-potentials.

We shall later use the exact GR formalism. But the above pseudo-potential approach mimic almost same sort of motion unless we consider very near to the blackhole horizon.

Space-Time Property

The Euler equation becomes

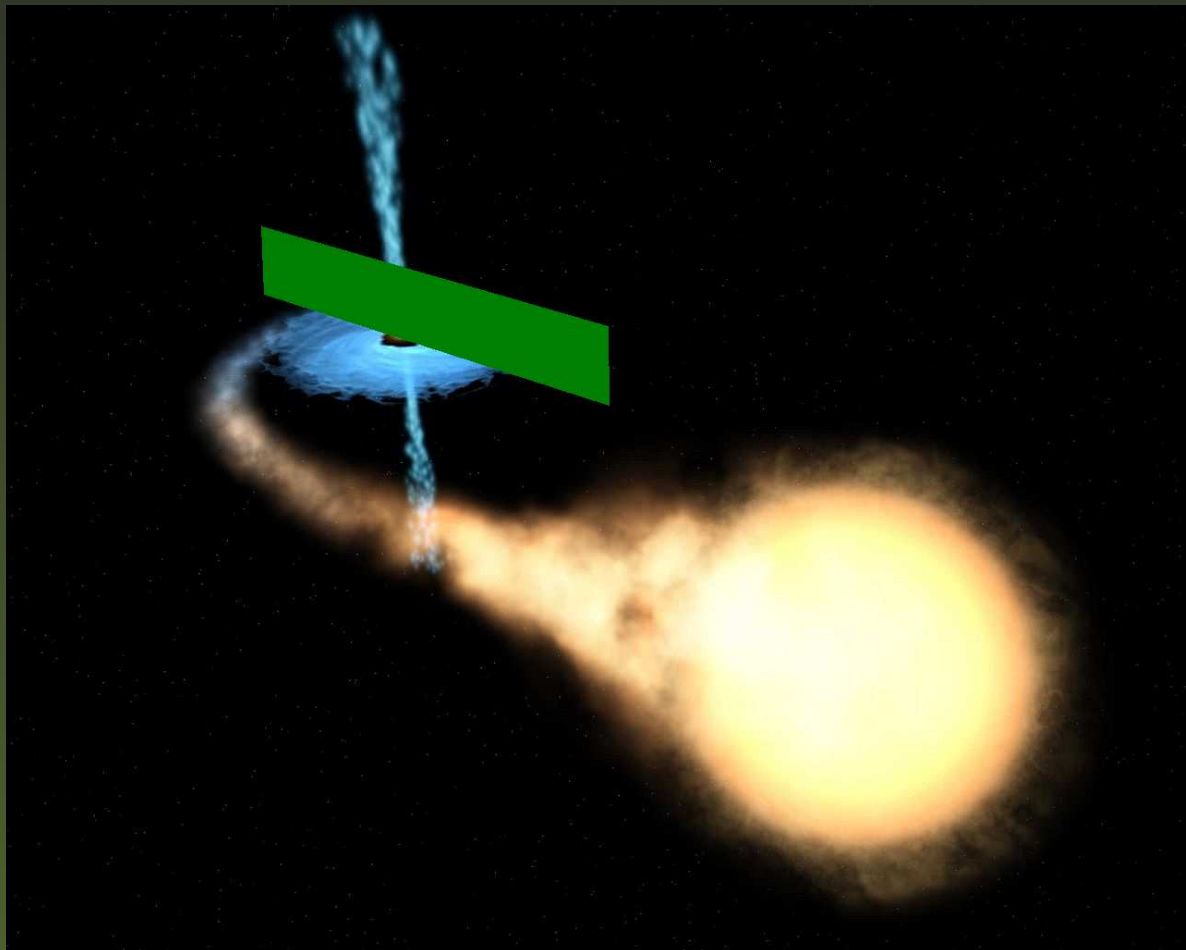
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \phi'(r) - \frac{\lambda^2}{r^3} = 0$$

where the Equation of State is

$$P = K\rho^\gamma.$$

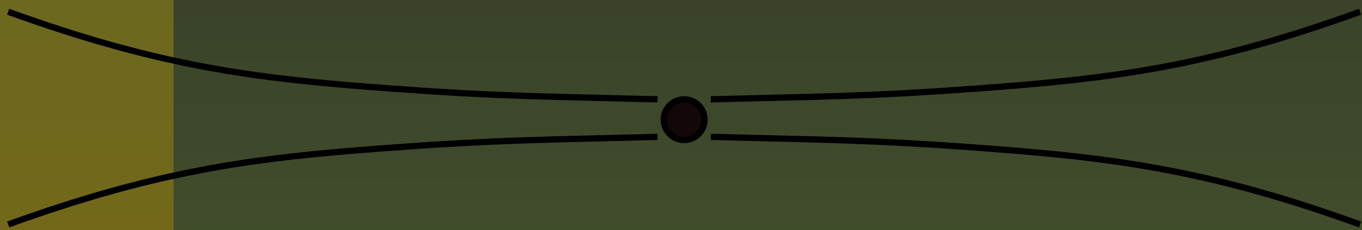
The λ is the specific angular momentum and γ is called the polytropic index

Disk Models

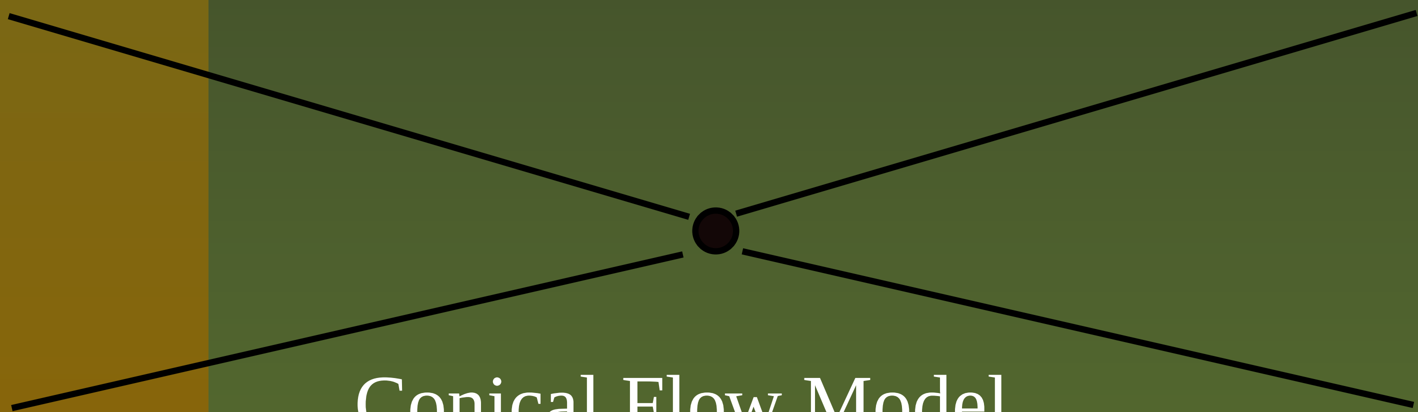


Disk Models

Constant Height Model



Vertical Equibrium Model



Conical Flow Model

Disk Models

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For the second case the disk height H is being estimated considering the hydrostatic equilibrium along the vertical direction and assuming the disk to be thin

$$H = c_{\mathrm{s}} \left(\frac{r}{\gamma \phi'} \right)^{1/2}$$

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For the third case $H \approx \alpha r$.

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where σ is equal to $(\gamma + 1)/2$ for the second case and equal to unity for the others. $g(r)$ takes different powers of r for different models.

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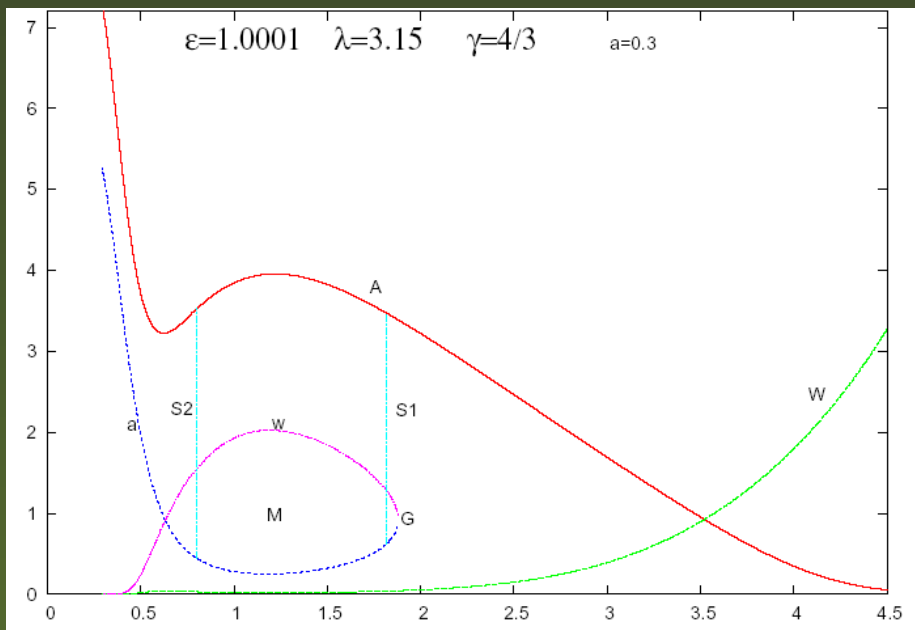
Thus we get two coupled nonlinear ordinary differential equations. The numerical solutions are possible and the typical solution looks like



Stationary Solutions

We are interested about the stationary solutions, i.e. for the solutions of Euler and Continuity equations when $\partial/\partial t$ terms vanish.
But the fixed point analysis technique borrowed from Dynamical System Analysis gives the nature of the solutions and may tell whether a the accretion is multi-transonic or not for a set of values of $\mathcal{E}$, λ and γ.
For example in conical flow we get,

$$2v^2\frac{dv^2}{dr} = \frac{2c_s^2/r + \lambda^2/r^3 - \phi'(r)}{v^2 - c_s^2}.$$

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The fixed point or critical point is the point where $\frac{dv^2}{dr}$ becomes $\frac{0}{0}$.

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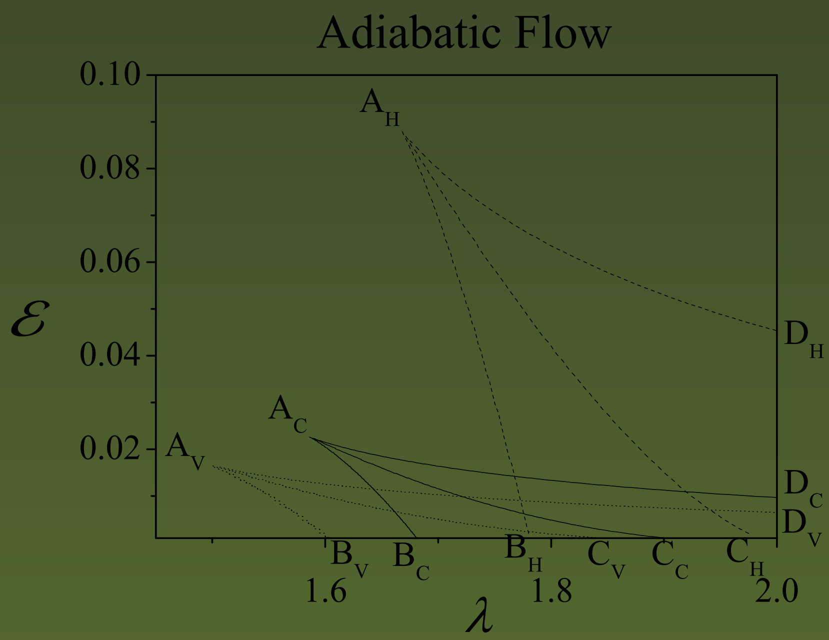
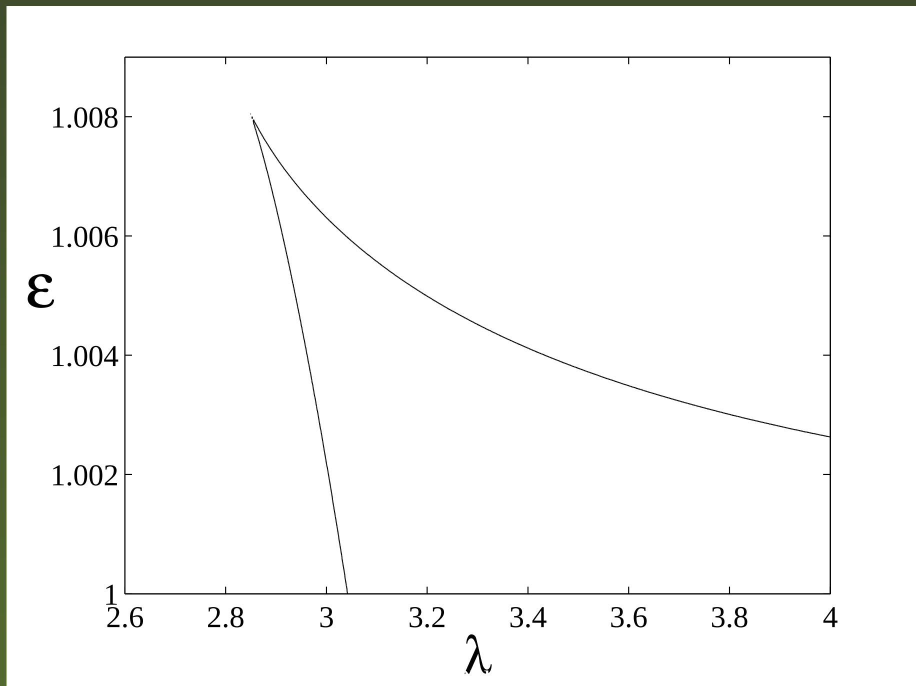
Application

In the case of pseudo Schwarzschild potentials, the equation delivering the critical point locations turns out to be of 4th degree polynomial which is analytically solvable. Making use of that we studied the comaparative behaviour among 3 models in $\mathcal{E} - \lambda$ space.



Application

In General Relativistic treatment of Schwarzschild Black Hole Accretion, similar procedure yields an 8th degree polynomial equation which is not analytically solvable. But still we identified the multicritical region in the $\mathcal{E} - \lambda$ space analytically by making use of the notion of bifurcation.



List of Collaborators

- Tapas K. Das (HRI, Allahabad)
- Rukmini Dey (HRI, Allahabad)
- Arnab K. Ray (Jaypee University,MP)
- Swagata Acharya (student, IIT, Kharagpur)
- Shilpi Agarwal (student, BHU, Varanasi)

The End

Thank You!