Model Dependence of Secular Instability in Black Hole Accretion Disc

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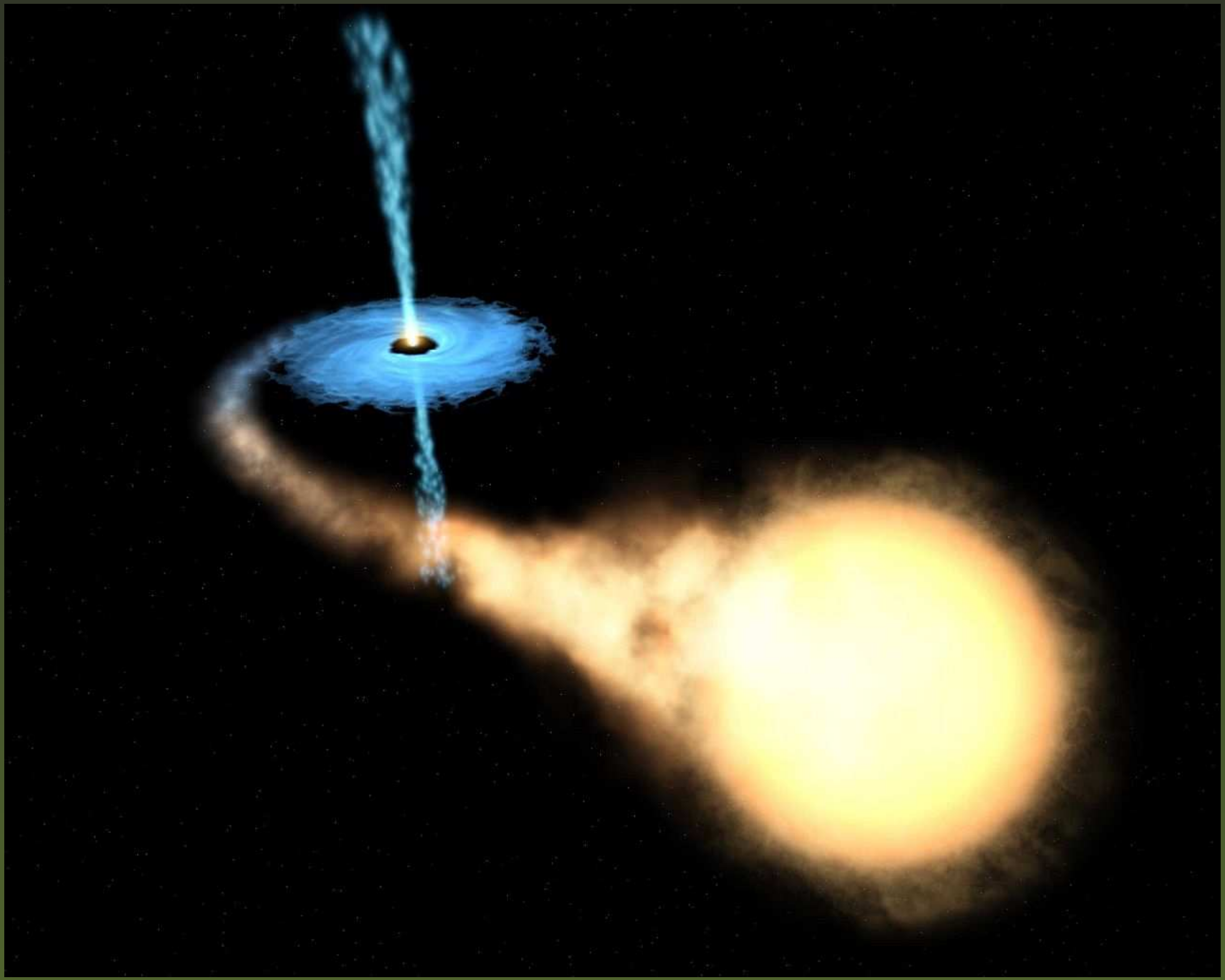
Accretion in Astrophysics



Accretion in Astrophysics

In our specific case:

- Observation: Black hole accretion discs in AGN.
- At large length scale the flow is stationary.
- Stability of such a stationary flow is a requirement.
- Accretion disc is investigated in quasi-viscous regime under pseudo-potential scheme.

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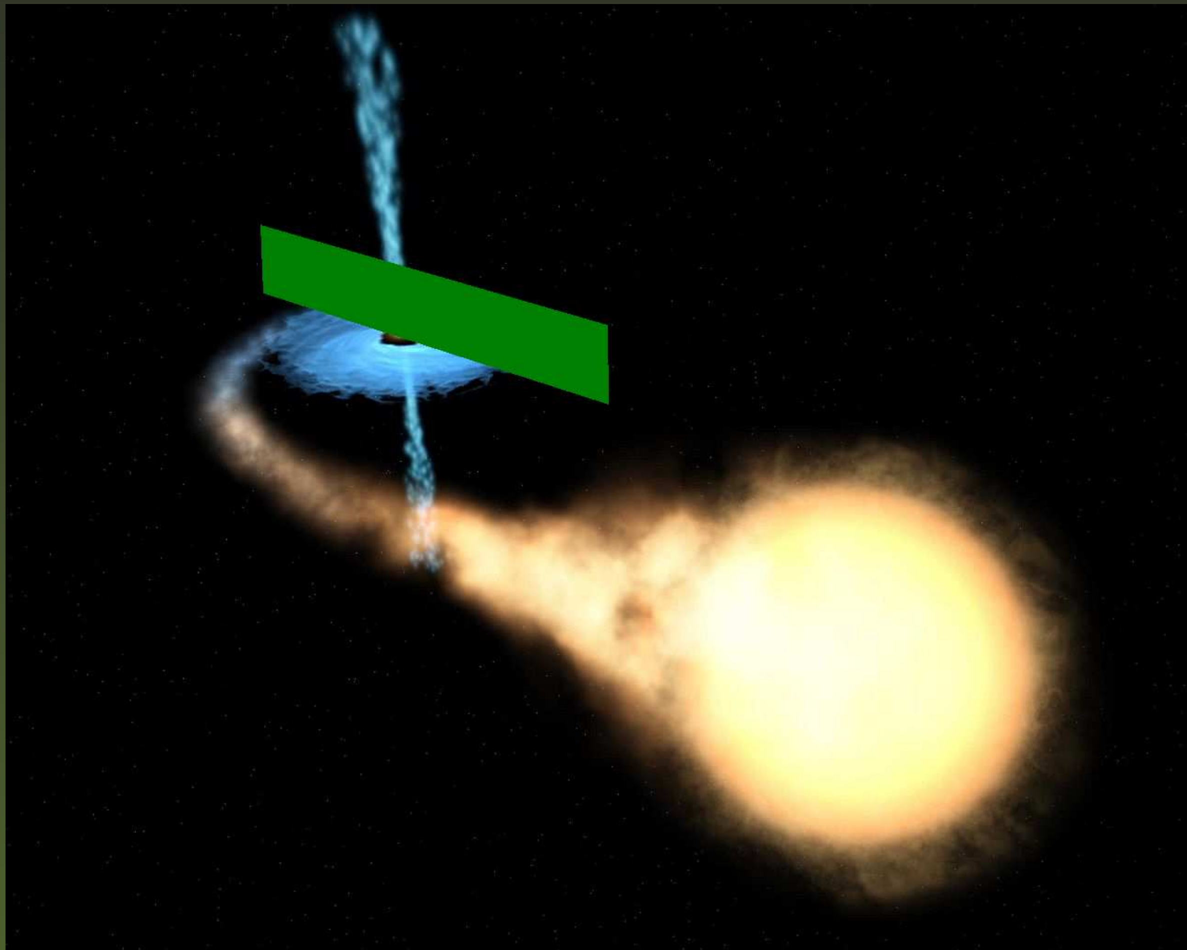
The disc models are

1. Constant height, i.e. $H = H_0$.
2. Conical Flow model, i.e. $H = \Theta r$.
3. Vertical Pressure equilibrium, $H = c_s \sqrt{\frac{r}{\gamma \phi'}}$.

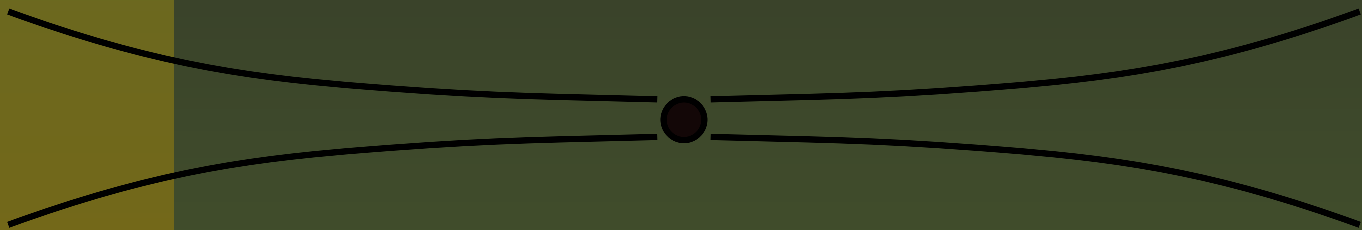
Disk Models



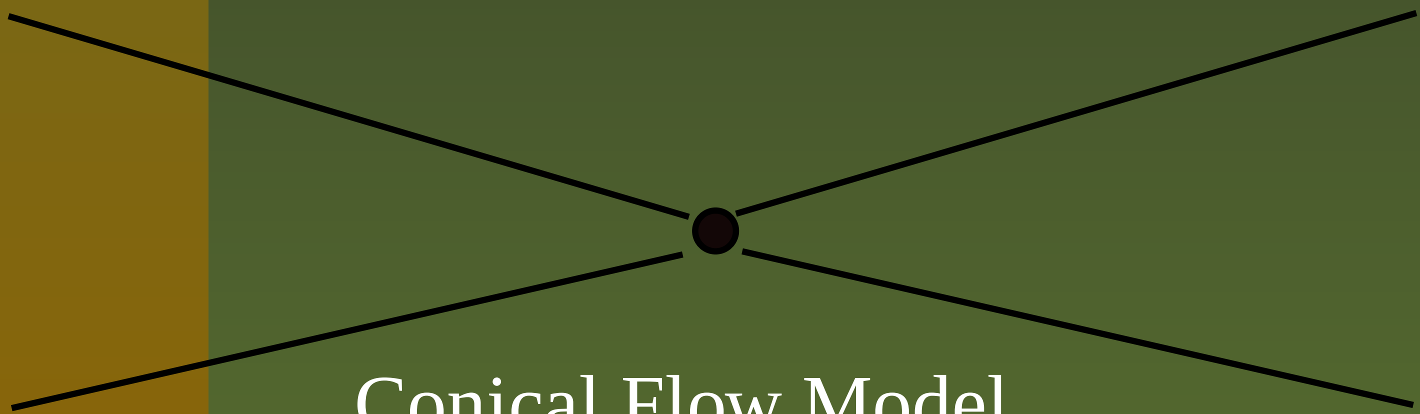
To parametrize $H = g(r)\rho^{\epsilon}$ where $\epsilon = 0$ for the first two and $\epsilon > 0$ for the rest one.

Disk Models

Constant Height Model



Vertical Equibrium Model



Conical Flow Model

Stationary Flow

- It is generally accepted that the accretion flow eventually reach stationary configuration.
- But for that to happen a necessary condition is: the stationary configuration must be stable under a time dependent perturbation.
- A number of authors [e.g. Nag et. al. (2011)] within last decade proved that this condition is obeyed at least in inviscid flow.
- But recently [e.g. Bhattacharjee et.al. (2009)]it was demonstrated that this is not the case at least in Vertical Equilibrium model even in quasi-viscous flow.

Motivation behind the Work

We wanted to investigate

- whether the result is dependent on disc models.
- what the characteristics length and time scale in such cases of instabilities.

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Assumptions made

- It is a low angular momentum flow.
- Viscosity is small so that its effect can be handled perturbatively (quasi-viscosity).
- The flow dynamics is governed by suitably chosen pseudo-potential $\phi(r)$ in a Newtonian scheme.

Governing Equations

Continuity equation,

$$\frac{\partial}{\partial t}(\Sigma) + \frac{1}{r}\frac{\partial}{\partial r}(\Sigma v r) = 0,$$

where Σ denotes mass per unit area of the disc, v denotes radial inward velocity. Stationarity: $f = \rho v r H$, a const.

Angular Momentum Balance Equation,

$$\rho H \frac{\partial}{\partial t}(r^2\Omega) + \rho v H \frac{\partial}{\partial r}(r^2\Omega) = \frac{1}{2\pi r}\left(\frac{\partial G}{\partial r}\right),$$

where, $G = 2\pi\nu\Sigma r^3 \frac{\partial \Omega}{\partial r}$ is the viscous torque with ν as kinematic viscosity.

Governing Equations

Radial Flow Equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \Phi'(r) - \frac{\lambda_{eff}^2}{r^3} = 0,$$

where, $\lambda_{eff} = r^2\Omega = \lambda_0 + \alpha r^2\tilde{\Omega}(c_s, v, r)$, for $\alpha = \nu c_s H$.
H varies from one disc model to another,
c_s being the sound speed.
Now, as $\alpha \ll 1$, it can be approximated as

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \Phi'(r) - \frac{\lambda_0^2}{r^3} - \frac{2\alpha\lambda_0 F(c_s, v, r)}{r^3} = 0$$

Time Dependent Perturbation

Consider small time dependent perturbation over the stationary solution of the previous equation as,

$$v(r,t) = v_0(r) + \tilde{v}(r,t)$$
$$\rho(r,t) = \rho_0(r) + \tilde{\rho}(r,t)$$
$$f(r,t) = f_0(r) + \tilde{f}(r,t)$$
$$F(r,t) = F_0(r) + \tilde{F}(r,t)$$

Time Dependent Perturbation

Retaining the the linear perturbing terms only one gets:

$$\frac{\partial}{\partial t}\left[\frac{v_0}{f_0}\frac{\partial \tilde{f}}{\partial t}\right] + \frac{\partial}{\partial t}\left[\frac{v_0^2}{f_0}\frac{\partial \tilde{f}}{\partial r}\right] + \frac{\partial}{\partial r}\left[\frac{v_0^2}{f_0}\frac{\partial \tilde{f}}{\partial t}\right] + \frac{\partial}{\partial r}\left[\frac{v_0}{f_0}\left(v_0^2 - \frac{c_{s0}^2}{1+\varepsilon}\right)\frac{\partial \tilde{f}}{\partial r}\right] - \frac{2\alpha\lambda_0^2}{r^3}\frac{\partial \tilde{F}}{\partial t} = 0$$

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when ϵ is a disc model dependent parameter,

$$\frac{\partial \tilde{F}}{\partial t} = \frac{c_{s0}}{\rho_0 v_0 r^2}\frac{(1+\gamma+4\varepsilon)}{(1+\varepsilon)}\frac{\partial \tilde{f}}{\partial r} + 2\int \frac{\partial}{\partial r}\left(\frac{c_{s0}}{\rho_0 v_0^2 r^2}\right)\frac{\partial \tilde{f}}{\partial t}dr$$

equation of state is given by $P = K\rho^\gamma$

Stationary Wave Solution

For stationary wave solution

$$\tilde{f}(r,t) = g_\omega(r)e^{-i\omega t}$$

Upon integrating we get finally,

$$A\omega^2 + B\omega + C = 0$$

Stationary Wave Solution

For stationary wave solution

$$\tilde{f}(r,t) = g_\omega(r) e^{-i\omega t}$$

$$A = \int v_0 g_\omega^2 dr \quad B = -4i\alpha\lambda_0^2 \int \frac{g_\omega f_0}{r^3} \left[\int g_\omega \frac{d}{dr} \left(\frac{c_{s0}}{\rho_0 v_0^2 r^2} \right) \right.$$

$$C = \int v_0 \left(v_0^2 - \frac{c_{s0}^2}{1+\varepsilon} \right) \left(\frac{dg_\omega}{dr} \right)^2 dr$$

$$+2\alpha\lambda_0^2 \int \frac{c_{s0} f_0}{\rho_0 v_0 r^5} \left(\frac{1+\gamma+4\varepsilon}{1+\varepsilon} \right) g_\omega \frac{dg_\omega}{dr} dr$$

Stationary Wave Solution

For stationary wave solution

$$\tilde{f}(r,t) = g_\omega(r)e^{-i\omega t}$$

It turns out

$$Re(-i\omega) \approx \alpha\xi(r)$$

Asymptotical behaviour at large r,

For Constant Height disc, $\xi(r) \sim r^{-2}$

For Conical Flow disc, $\xi(r) \sim r^{1}$

For Vertical Equilibrium disc, $\xi(r) \sim r^{5/2}$

Travelling Wave Solution

Rearrangement of terms in the original equation gives

$$P\frac{d^2 g_\omega}{dr^2} + Q\frac{dg_\omega}{dr} - Rg_\omega + T\int g_\omega \frac{d\sigma}{dr}dr = 0$$

Travelling Wave Solution

$$P = v_0^2 - \frac{c_{s0}^2}{1+\varepsilon}$$

$$Q = \frac{1}{v_0}\frac{d}{dr}\left[v_0\left(v_0^2 - \frac{c_{s0}^2}{1+\varepsilon}\right)\right]$$

$$- 2i\omega v_0 - \frac{2\alpha\lambda_0^2 c_{s0} f_0}{\rho_0 v_0^2 r^5}\left(\frac{1+\gamma+4\varepsilon}{1+\varepsilon}\right)$$

$$R = 2i\omega\frac{dv_0}{dr} + \omega^2$$

$$T = \frac{4\alpha\lambda_0^2 i\omega}{v_0 r^3}, \quad \sigma = \frac{c_{s0} f_0}{\rho_0 v_0^2 r^2}$$

Travelling Wave Solution

Taking the spatial part of the perturbation as

$$g_\omega(r) = exp(i\omega s(r))$$

Travelling Wave Solution

Taking the spatial part of the perturbation as

$$g_\omega(r) = exp(i\omega s(r))$$

$$s(r) = \sum_{n=0}^{\infty} \frac{k_n(r)}{\omega^n}$$

where $\frac{1}{\omega}$ is very very small.
If the imaginary part of this series converges rapidly for all r, one may safely argue that the perturbation always remains finite.

Travelling Wave Solution

Taking the spatial part of the perturbation as

$$g_\omega(r) = exp(i\omega s(r))$$

Making coefficient of $\omega^2 = 0$ gives

$$k_0 = \int \frac{1}{v_0 \pm \frac{c_{s0}}{\sqrt{1+\varepsilon}}} dr$$

Travelling Wave Solution

Taking the spatial part of the perturbation as

$$g_\omega(r) = exp(i\omega s(r))$$

But making coefficient of $\omega = 0$,

$$ik_1 = -\frac{i}{2} ln\left(\frac{v_0 c_{s0}}{\sqrt{1+\varepsilon}}\right)$$
$$\pm\, i \int \alpha \lambda_0^2 c_{s0}^2 f_0 \left(\frac{1+\gamma+4\varepsilon}{\sqrt{1+\varepsilon}}\right)\left(v_0 \mp \frac{c_{so}}{\sqrt{1+\varepsilon}}\right)^{-1} dr$$

Thus asymptotically k_1 diverges as $r \to \infty$ if $\alpha \neq 0$.

References

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3. Bilic, N., Choudhury A., Das T.K. and Nag, S. *Class. Quant. Grav.*, 31:035002, 2013.

Thank You !